Call Completion Probability in Heterogeneous Networks with Energy Harvesting Base Stations

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Abstract—In this paper, we analyse the call completion probability in a two-tier heterogeneous network (HetNet), where all base stations (BSs) in each tier are powered solely by energy harvesting. Since energy harvesting BSs may need to be kept OFF and allowed to recharge, users connected to a BS that turns OFF need to be served by neighboring BSs that are ON. This hand-off of a call can impact the call performance from a users point of view. We formulate the call completion probability for HetNets by adapting the definition from traditional cellular networks. Adopting a realistic BS energy consumption model and using tools from stochastic geometry, we derive very tight upper and lower bounds on the call completion probability in the presence of Rayleigh fading, interference and energy harvesting BSs. We examine the impact of the system parameters on the completion probability. The results show that the macro BS energy harvesting parameters have the dominant impact on the call completion probability. In particular, the call completion probability is an increasing function of the macro BS battery capacity and the minimum energy level at which macro BS switches back ON. However, it is an increasing function of the macro BS energy harvesting rate only when the macro BS battery capacity and the minimum energy level at which macro BS switches back ON are large. The results can be used in network planning to ensure certain quality-of-service (QoS) to users in terms of call completion probability.

Index Terms—Heterogeneous networks, energy harvesting base stations, call completion probability, coverage probability.

I. Introduction

Base stations that can harvest renewable energy sources, such as solar and wind, are being gradually deployed throughout the world, e.g., China Mobile has established 1000+ BSs in Tibet about 80% of which are powered by solar energy [1] and Nokia-Siemens has built 'green' (i.e., renewable energy powered) BS sites in Germany [2]. This shows that it is technically feasible to have self-powered BSs. Powering BSs with renewable energy is attractive from an environmental as well as economic perspective, since BSs account for more than 50% of the energy consumption in wireless networks [3, 4]. It can be expected that regulatory pressure for greener techniques in developed countries and the lack of a dependable electrical grid in many Asian and African countries will drive the future push towards self-powered BSs [5].

In the context of energy-efficient communications, heterogeneous networks (HetNets) where small cells are deployed in existing macrocells have become a promising 5G technology. The small cell BSs in HetNets have smaller transmit power, compared to conventional macro BSs, e.g., 50W, 2W and 0.2W for macro, pico and femto BSs in LTE [6]. The smaller power requirement of small cell BSs make them especially attractive for being powered by renewable energy sources, opening up the possibility of low cost "drop and play" deployment [7].

Recently there has been a lot of interest in BSs powered by renewable energy sources. For *traditional cellular network*, the feasibility of powering macro BSs in LTE with only renewable energy sources is studied in [5]. Solar radiation models for cloudless and cloudy days and BS energy allocation algorithm are proposed in [1]. Frameworks for BS planning in cellular networks, where BSs are powered by both on-grid energy and renewable energy, are considered in [2, 8–10], which aim to minimize the total energy consumption and/or to balance the traffic load. These frameworks enhance traditional BS planning tools [11] which do not take renewable energy into account. The joint problem of BS sleeping (i.e., turning BS's OFF if it has light load to save energy) and resource allocation is investigated in [12] where BSs are assumed to be powered jointly by a grid and renewable energy. Considering a mixed deployment of renewable energy and grid powered BSs, energy-harvesting aware sleep control for cellular systems is proposed in [13].

For *HetNets*, there have been only a few works considering energy harvesting BSs. The pioneering work in [14] proposed a K-tier HetNet model where BSs across different tiers are powered by renewable energy. Using tools from stochastic geometry, the uncertainty in the availability of BSs in each tier due to finite battery capacity and randomness in the energy harvesting process is characterized. The problem of adaptive user association in HetNets with renewable energy powered BS is considered in [7], while energy efficient planning in a 2-tier HetNet is considered in [15]. Note that most of the prior works [1, 7–9, 15] use tools from optimization technique, except for [14] which uses stochastic geometry. However, the focus in [14] is BS availability analysis. To the best of our knowledge, the effect of the energy harvesting BSs on the user's call performance is not considered.

In this paper, we use stochastic geometry to analyse the call performance in HetNets with energy harvesting BSs. We modify and adopt the call completion probability as a metric for call performance analysis in HetNets. Since the harvesting of renewable energy is a highly dynamic process, BSs powered solely by renewable energy may need to be intermittently

turned OFF to recharge. This means that all the BSs in the network may not always be available to serve users and the users being served by a BS which turns OFF need to be offloaded to nearby BSs, which impacts the call performance. In this context, we make the following contributions:

- We derive tight upper and lower bounds for the call completion probability in a 2-tier HetNet with energy harvesting BSs in the presence of Rayleigh fading channels and interference. In order to determine the call completion probability, we derive expressions for the mean BS ON and OFF time, respectively, using a realistic BS energy consumption model [16]. Our results include previous BS availability results in [14] as a special case.
- We analyse the impact of the system parameters on the call completion probability. We show that micro BS energy harvesting parameters have a minor impact while the macro BS energy harvesting parameters have the dominant impact on the call completion probability. In particular, the call completion probability is an increasing function of the macro BS battery capacity and the minimum energy level at which macro BS switches back ON. However, it is an increasing function of the macro BS energy harvesting rate only when the macro BS battery capacity and the minimum energy level at which macro BS switches back ON are large.

The rest of this paper is organized as follows. Section II presents the system model and the known results from stochastic geometry. Section III derives the call completion probability and the BS availability results. Section IV presents the results. Finally, Section V provides the conclusions.

II. SYSTEM MODEL

We consider the downlink of a $K = 2$-tier HetNet, where tier 1 is modelled as macrocell and tier 2 as microcell. For notational simplicity, we assign subscript $k = 1, 2$ for macrocell and microcell, respectively. The location of macro BSs, micro BSs and users are modelled by an independent Poisson Point Process (PPP) with constant densities λ_1, λ_2 and λ_u, respectively. The BSs in each tier are allocated the same constant transmit power P_1 and P_2, respectively, and share the same frequency spectrum using orthogonal frequency division multiple access (OFDMA). We consider a path loss and Rayleigh fading channel model. We assume the system is interference-limited, i.e., the thermal noise is negligible compared to the interference power. Thus, each user is associated with the BS in the k-tier that provides the highest long term received power. The target throughput threshold T at the users is the same for both tiers.

A. BS Energy Harvesting Model

We assume that all BSs are solely powered by renewable energy, i.e., each BS has its own energy harvesting module as well as an energy storage device of finite capacity N_k. The energy arrival process at each tier BS is modelled as a Poisson process with mean energy harvesting rate μ_k [14].

B. BS Energy Utilization Model

According to [16], the energy utilization of BSs can be modelled by two parts: (i) static energy utilization which corresponds to the energy utilization without any traffic load and (ii) dynamic energy utilization which is related to the traffic load (i.e., the number of users served by a BS). Hence, we model the BS energy utilization rate as

$$\gamma_k = S_k + v_k = S_k + D_k \overline{A_k} \lambda_u, \tag{1}$$

where the constant S_k represents the static energy utilization part, and $v_k = D_k \overline{A_k} \lambda_u$ denotes the mean dynamic energy utilization part, in which the constant D_k is the dynamic energy utilization rate per user, $\overline{A_k}$ is the kth tier BS's average service area and λ_u is the user density.

Remark 1. In [14], a simplified energy utilization model is considered assuming $S_k = 0$ and $D_k = 1$, which is a special case of the general model considered in this paper.

C. BS Operational Model

Following [14], we assume that each BS transmits to its users in each resource block over a short time scale, while each BS harvests energy over a long time scale. This facilitates tractability and allows the BS energy state, J_k, to be modelled as a continuous-time Markov chain, with birth rate μ_k (i.e., mean energy harvesting rate) and death rate v_k (i.e., mean dynamic energy utilization rate). Since all BSs are solely powered by renewable energy, at any given time, each BS can be in one of two operational modes: ON and OFF. During OFF mode, we assume a BS consumes a negligible amount of power. During ON mode, a BS harvests energy and serves users at the same time (i.e., energy is consumed). In this work, we assume that:

1) Each BS decides independently whether to be ON or OFF based on its instantaneous energy level.
2) BS temporarily turns OFF if its energy state reaches 0.
3) BS turns ON after sometime when its energy state reaches $N_k^c (< N_k)$ through energy harvesting, where N_k^c denotes the minimum energy level at which BS switches back ON.

D. Summary of Relevant Results from Stochastic Geometry

It has been shown in [17] that, when each user is associated with the BS that provides the highest long term received power and the target throughput threshold at user is the same for both tiers, the probability that a typical user is associated with the kth tier BS is given by

$$\mathcal{P}_k^s = \lambda_k \mathcal{P}_k^a \overline{A_k}, \tag{2}$$

where the average service area of a kth tier BS is given by [14]

$$\overline{A_k} = \frac{P_k^{\frac{2}{\alpha}}}{\sum_{j=1}^2 \lambda_j \mathcal{P}_j^a P_j^{\frac{2}{\alpha}}}, \tag{3}$$

where α is the pathloss exponent and $\mathcal{P}_k^{\mathrm{a}}$ represents the availability of kth tier BS, which is defined as [14]

$$\mathcal{P}_k^{\mathrm{a}} = \frac{\mathbb{E}[J_k^{ON}]}{\mathbb{E}[J_k^{ON}] + \mathbb{E}[J_k^{OFF}]}. \tag{4}$$

where $\mathbb{E}[J_k^{ON}]$ and $\mathbb{E}[J_k^{OFF}]$ represent the mean kth tier BS ON and OFF times and $\mathbb{E}[\cdot]$ denotes expectation.

For a $K = 2$-tier HetNet, by adapting the results in [18] with effective BS density $\lambda_1' = \mathcal{P}_1^{\mathrm{a}}\lambda_1$ and $\lambda_2' = \mathcal{P}_2^{\mathrm{a}}\lambda_2$, the coverage probability that the signal to interference ratio (SIR) of a random user is greater than a threshold T is given by

$$\mathcal{P} = 2^{-\frac{T}{B}} \left[\left(\frac{\frac{\epsilon\lambda_1' + \lambda_2'}{\epsilon} K_1}{(1 - 2^{-\frac{T}{B}})\lambda_u + \frac{\epsilon\lambda_1' + \lambda_2'}{\epsilon} K_1} \right)^{K_1} \mathcal{P}_1^{\mathrm{s}} + \right.$$
$$\left. \left(\frac{(\epsilon\lambda_1' + \lambda_2')K_2}{(1 - 2^{-\frac{T}{B}})\lambda_u + (\epsilon\lambda_1' + \lambda_2')K_2} \right)^{K_2} \mathcal{P}_2^{\mathrm{s}} \right], \tag{5}$$

where B is the bandwidth, $\alpha = 4$ and $\epsilon = \left(\frac{P_1}{P_2}\right)^{\frac{2}{\alpha}} = 10^{0.5} = 3.1632$ is a function of the ratio of the transmit power of macro BS to micro BS. K_1 and K_2 are found using kernel density estimation as $K_1 = 3.575 \frac{\lambda_1' + 0.4106\lambda_2'}{\lambda_1' + 0.1673\lambda_2'}$, $K_2 = 3.575 \frac{\lambda_2' + 2.5327\lambda_1'}{\lambda_2' + 5.1952\lambda_1'}$ [18].

III. CALL COMPLETION PERFORMANCE ANALYSIS

In this section, we derive the call completion probability to evaluate the HetNet performance with energy harvesting BSs.

A. Definitions and Formulation

For a traditional cellular network, corresponding to $K = 1$-tier, the call completion probability can be defined as follows [19].

Definition 1. (Call completion probability for cellular networks) The call completion probability is the probability that a new call is successfully connected to the cellular network and is not dropped due to handoff failure or link breakage until the user ends the call. Mathematically it can be expressed as [19]

$$\mathcal{P}^{\mathrm{c}} = (1 - \rho_0)\mathbb{P}\left(T_{\mathrm{c}} < \left(R + \sum_{i=2}^{N} T_i\right), T_{\mathrm{c}} < \sum_{i=1}^{M} V_i\right) \tag{6}$$

$$= (1 - \rho_0)\mathcal{M}_{T_{\mathrm{c}}}(-(a+b)), \tag{7}$$

where $\mathbb{P}(\cdot)$ denotes probability, ρ_0 is the probability that a new call is blocked, T_{c} is the call holding time of a user, N is the index of the last cell where the user ends the call, T_i is the ith cell dwell time (independent and identically distributed T), R is the residual life of the call in the first cell, V_i is the ith good link period, M is the index of the last good link where the user ends the call, $a = \frac{\rho_f}{\rho_f \mathbb{E}[R] + (1 - \rho_f)\mathbb{E}[T]}$, $b = \frac{\rho_{\mathrm{link}}}{\mathbb{E}[V]}$, ρ_f is the probability of handoff failure, ρ_{link} is the probability of a link breakage, $\mathbb{E}[R]$ is the mean residual life of the call in the first cell which depends on the distribution of T, $\mathbb{E}[V]$ is the expected time of a good link period, $\mathbb{E}[T]$ is the mean cell dwell time and $\mathcal{M}_{T_{\mathrm{c}}}(t) = \int_0^{\infty} \exp(tx) f_{T_{\mathrm{c}}}(x)dx$ is the

moment generating function (MGF) of the random variable T_{c}.

We adapt Definition 1 to formulate the call completion probability for HetNets with energy harvesting base stations as follows.

1) In (7), it is implicit in the formulation that the user mobility causes the hand-off between BSs. In this work, we assume that as long as a user is connected to a BS, it does not move outside the coverage area of the BS, i.e., user mobility does not trigger a hand-off and only BSs turning ON and OFF causes the hand-off.

2) As we are considering a two-tier HetNet, cross tier hand-offs can occur. Under this assumption, the mean cell dwell time $\mathbb{E}[T]$ in term a in (7) is determined not only by the expected ON time for a BS $\mathbb{E}[J_k^{ON}]$ but also by the association probability $\mathcal{P}_k^{\mathrm{s}}$ in (2) as (see Appendix A)

$$\mathbb{E}[T] = \mathcal{P}_1^{\mathrm{s}}\mathbb{E}[J_1^{ON}] + \mathcal{P}_2^{\mathrm{s}}\mathbb{E}[J_2^{ON}]. \tag{8}$$

3) For $\mathbb{E}[R]$ in term a in (7), it has been shown in [19] that $\mathbb{E}[R] = \mathbb{E}[T^2]/2\mathbb{E}[T]$. It is not easy to obtain the distribution of T in our case. However, from its definition, since R is bounded as $0 \le R < T$, we can bound $\mathbb{E}[R]$ by 0 and $\mathbb{E}[T]$. The tightness of these bounds will be demonstrated later in the results.

4) For tractability, we assume an exponential distribution for the call holding time T_c, which is a commonly used model in the literature [19].

5) Finally, since we wish to focus on the effect of the energy harvesting base stations, we assume that a call initiated by the user is never blocked by the network, i.e., $\rho_0 = 0$.

Using the above assumptions, we can express the bounds on $\mathcal{P}^{\mathrm{c}}$ in terms of the mean kth tier BS ON time. This result is stated in the lemma below.

Lemma 1. *For a $K = 2$-tier HetNet with energy harvesting BS, assuming the call initiated by the user is never blocked by the network, the call holding time follows an exponential distribution and hand-off is only caused by BSs turning ON and OFF, the overall call completion probability is bounded by*

$$\frac{1}{M_{T_{\mathrm{c}}}(a_1 + b) + 1} \le \mathcal{P}^c \le \frac{1}{M_{T_{\mathrm{c}}}(a_2 + b) + 1}, \tag{9}$$

where $M_{T_{\mathrm{c}}}$ is the average call holding time, $a_1 = \frac{\rho_f}{(1 - \rho_f)\left(\mathcal{P}_1^{\mathrm{s}}\mathbb{E}[J_1^{ON}] + \mathcal{P}_2^{\mathrm{s}}\mathbb{E}[J_2^{ON}]\right)}$, $a_2 = \frac{\rho_f}{\mathcal{P}_1^{\mathrm{s}}\mathbb{E}[J_1^{ON}] + \mathcal{P}_2^{\mathrm{s}}\mathbb{E}[J_2^{ON}]}$, $b = \frac{\rho_{\mathrm{link}}}{\mathbb{E}[V]}$, $\mathbb{E}[J_k^{ON}]$ is the mean kth tier BS ON time and $\mathcal{P}_k^{\mathrm{s}}$ is the kth tier association probability given in (2).

Proof: See Appendix A. ■

As shown in (9), the call completion probability is strongly dependent on the mean kth tier BS ON time as well as the BS availability (which depends on both mean kth tier BS ON and OFF times). The values of these parameters are derived in the following subsection.

B. BS Availability Analysis

In this section, we derive the BS availability results taking into account the accurate BS energy utilization model in (1). The main results are presented in the two lemmas below.

Lemma 2. *The mean time a kth tier BS spends in the ON state is given by the solution of the following equation:*

$$\mathbb{E}[J_k^{ON}] = \frac{\left(\frac{\mu_k}{v_k}\right)^{N_k} - \left(\frac{\mu_k}{v_k}\right)^{N_k - N_k^c + S_k \mathbb{E}[J_k^{ON}]}}{(v_k - \mu_k)^2 \mu_k^{-1}} - \frac{N_k^c - S_k \mathbb{E}[J_k^{ON}]}{\mu_k - v_k},$$

$$(10)$$

where μ_k is the mean energy harvesting rate, v_k is the mean dynamic energy utilization rate, N_k is the battery capacity, N_k^c is the minimum energy level at which BS switches back ON and S_k is the static energy utilization rate.

Proof: See Appendix B. ■

Lemma 3. *The mean time a kth tier BS spends in the OFF state is given by*

$$\mathbb{E}[J_k^{OFF}] = \frac{N_k^c}{\mu_k}. \qquad (11)$$

Proof: During OFF time, the BS does not consume any energy. Hence the change in the energy state can be modelled as a pure birth process and the mean OFF time is the time required to harvest N_k^c units of energy. This can be found as the sum of N_k^c exponentially distributed random variables with mean $1/\mu_k$ [14]. Hence, we arrive at (11). ■

Remark 2. (10) is a generalisation of the result in [14]. If we assume $S_k = 0$, $D_k = 1$ and $N_k^c = 1$, (10) simplifies to [14, Eq. (26)]. Also, (10) is valid for any α, unlike the coverage result in (5) which is only valid for $\alpha = 4$. While (10) does not have a closed-form in general, it can be easily solved numerically.

Substituting (10) and (11) in (4), the BS availability can be determined. Also substituting (10) and (4) in (9), the call completion probability for a HetNet with energy harvesting BSs can be determined.

IV. RESULTS AND DISCUSSION

In this section, we present the results for the call completion probability in (9) for a $K = 2$ tier HetNet with energy harvesting BSs.

According to [20], the overall energy utilization of a macro BS only varies about 3% for a 3G BS and 2% for a 2G BS over a period of several days, while its traffic load varies from no load to peak load. Thus, for the purpose of generating the results, we ignore the dynamic energy utilization of a macro BS and set $D_1 = 0$. Thus, we use

$$\gamma_1 = S_1, \qquad (12)$$
$$\gamma_2 = S_2 + v_2. \qquad (13)$$

For the macro BS, the expected ON time can therefore be solved using (10) as

$$\mathbb{E}[J_1^{ON}] = \frac{N_1^c}{S_1 - \mu_1}. \qquad (14)$$

TABLE I
COMMONLY USED SYSTEM PARAMETER VALUES.

Parameters	Notation	Value
Path loss exponent	α	4
Ratio of transmit power	P_1/P_2	10
Macro BS density (per km^2)	λ_1	1.4
Micro BS density (per km^2)	λ_2	3.7
User density (per km^2)	λ_u	18
Static energy utilization rate of micro BSs (kJ/s)	S_2	0.025
Dynamic energy utilization rate of micro BSs (kJ/s)	D_2	0.007
Static energy utilization rate of macro BSs (kJ/s)	S_1	1.43
Dynamic energy utilization rate of macro BSs (kJ/s)	D_1	0
Average call duration (s)	M_c	180
Handoff failure probability	ρ_f	0.01
Link breakage probability	ρ_{link}	0.01
Expected good link period (s)	$\mathbb{E}[V]$	60
Bandwidth (MHz)	B	20
Energy harvest rate of macro BSs (kJ/s)	μ_1	1.3
Energy harvest rate of micro BSs (kJ/s)	μ_2	0.2
Battery capacity of macro BSs (kJ)	N_1	20
Battery capacity of micro BSs (kJ)	N_2	2
Ratio of switching state and battery capacity	N_k^c/N_k	0.5

This shows that the expected ON time for macro BS only relies on the minimum energy level at which macro BS switches back ON, macro BS static energy consumption and macro BS energy harvesting rate. Substituting (14) and (11) in (4), the macro BS availability can be simplified to

$$\mathcal{P}_1^a = \frac{\mu_1}{S_1}. \qquad (15)$$

This shows that the macro BS availability is independent of the macro BS battery capacity and the minimum energy level at which macro BS switches back ON.

Unless specified otherwise, the values of the commonly used parameters are summarized in Table I. These values are chosen consistent with [14, 18–20] to ensure that the coverage probability in (5) is around 0.9. As shown in [6], the coverage probability is insensitive to the change in the BS densities in an interference-limited HetNet. Hence, varying the number of BSs in the ON/OFF states has marginal effect on the coverage probability. In contrast, the expected ON time directly affects the call completion probability. In what follows, we investigate the effects of three important energy-related parameters on the call completion probability.
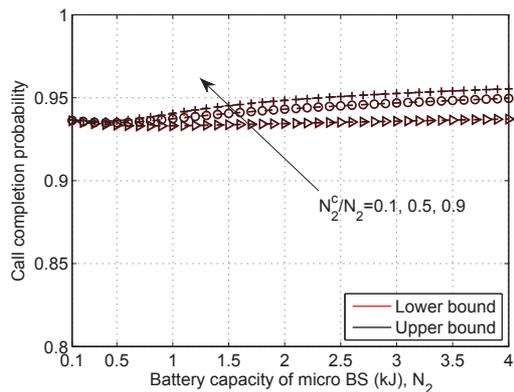
A. Effect of Battery Capacity

Fig. 1 plots the call completion probability versus the battery capacity for macro BS and micro BS, respectively, with different ratio of the minimum energy level at which BS switches back ON and battery capacity (i.e., $N_k^c/N_k = 0.1, 0.5, 0.9$) for $k = 1, 2$. The curves are plotted using (9).

Fig. 1(a) shows that the call completion probability increases when either N_1 or N_1^c/N_1 increases. This can be explained in detail as follows. For the *macro BSs*, when either N_1 or N_1^c/N_1 increases, N_1^c becomes larger and from (14) we can see that it increases $\mathbb{E}[J_1^{ON}]$. However, $\mathbb{E}[J_1^{OFF}]$ in (11) also increases in proportion such that $\mathcal{P}_1^a$ in (15) is constant. For the *micro BSs*, since the values of N_2 and N_2^c/N_2 are
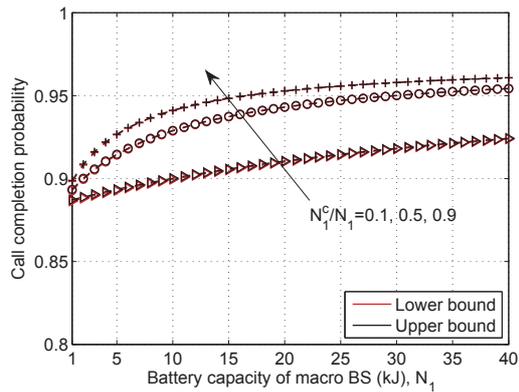
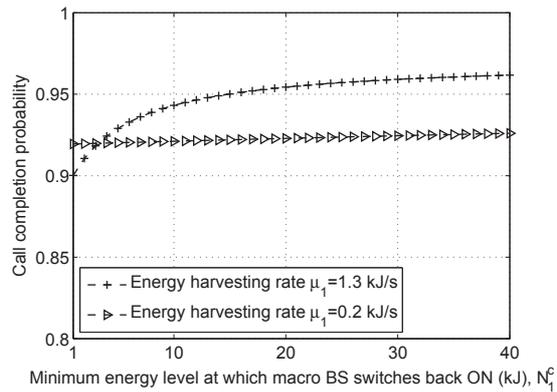(a) Effect of battery capacity for macro BSs.

(b) Effect of battery capacity for micro BSs.

Fig. 1. Call completion probability versus the battery capacity with different ratio of the minimum energy level at which BS switches back ON and the battery capacity.
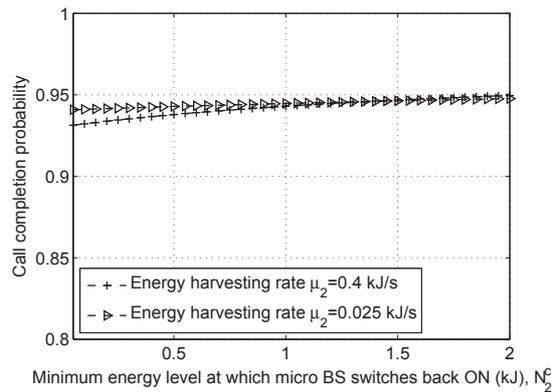

(a) Effect of the minimum energy level at which macro BS switches back ON.

(b) Effect of the minimum energy level at which micro BS switches back ON.

Fig. 2. Call completion probability versus the minimum energy level at which BS switches back ON, with different energy harvesting rate.

fixed in Fig. 1(a), $\mathbb{E}[J_2^{ON}]$, $\mathbb{E}[J_2^{OFF}]$ and $\mathcal{P}_2^{a}$ are constants. Since both $\mathcal{P}_1^{a}$ and $\mathcal{P}_1^{a}$ are constants, from (2) we can see that both $\mathcal{P}_1^{s}$ and $\mathcal{P}_2^{s}$ are also constant. Thus, in (8), all terms are constant except $\mathbb{E}[J_1^{ON}]$ which has the dominant effect. As $\mathbb{E}[J_1^{ON}]$ increases, it increases $\mathbb{E}[T]$ and from (9) we can that see that the call completion probability increases.

In contrast, Fig. 1(b) shows that the call completion probability is an non-increasing function of the micro BS's capacity. This can be explained in detail as follows. For the *micro BSs*, when N_2 increases, both $\mathbb{E}[J_2^{ON}]$ and $\mathbb{E}[J_2^{OFF}]$ increase. However, $\mathbb{E}[J_2^{ON}]$ increases faster compared to $\mathbb{E}[J_2^{OFF}]$, such that $\mathcal{P}_2^{a}$ is increasing. This is different from the previous case because $\mathbb{E}[J_1^{ON}]$ is determined using (14), while $\mathbb{E}[J_2^{ON}]$ is determined using (10). For the *macro BSs*, since the values of N_1 and N_1^{c}/N_1 are fixed in Fig. 1(b), $\mathbb{E}[J_1^{ON}]$, $\mathbb{E}[J_1^{OFF}]$ and $\mathcal{P}_1^{a}$ are constants. Consequently, from (2), we can see that $\mathcal{P}_1^{s}$ decreases and $\mathcal{P}_2^{s}$ increases. Thus, in (8), $\mathcal{P}_1^{s}$ is a decreasing function and $\mathbb{E}[J_1^{ON}]$ is a constant while $\mathcal{P}_2^{s}$ and $\mathbb{E}[J_2^{ON}]$ are increasing functions of N_2. The interaction of these effects results in the call completion probability first decreasing slightly and then increasing slightly.

The results in Fig. 1(a) and Fig. 1(b) illustrate that the upper

and lower bounds for the call completion probability almost overlap, which demonstrates the tightness of our derived bounds. In the following figures, for the sake of clarity, we only plot the call completion probability results using the upper bound.

B. Effect of the Minimum Energy Level at which BS switches back ON

Fig. 2 plots the call completion probability versus the minimum energy level at which BS switches back ON, for macro BS and micro BS respectively, with different energy harvesting rate. Fig. 2(a) shows that increasing N_1^{c} improves the call completion probability. As highlighted previously, when N_1^{c} increases the macro BS expected ON time increases, which has the dominant effect and causes the call completion probability to increase. Fig. 2(b) shows that when N_2^{c} increases, the call completion probability increases very slightly. This is because the the micro BS expected ON time increases while the other terms in (8) are either constant (i.e., the macro BS expected ON time, which far exceeds micro BS expected ON time) or virtually unchanged (i.e., both tier association probabilities). Hence, the call completion probability increases very slightly.
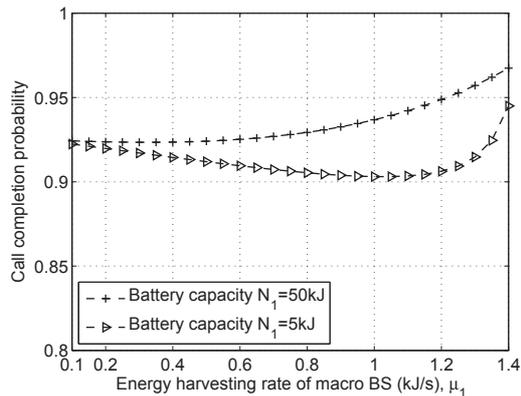
(a) Effect of energy harvesting rate for macro BSs.
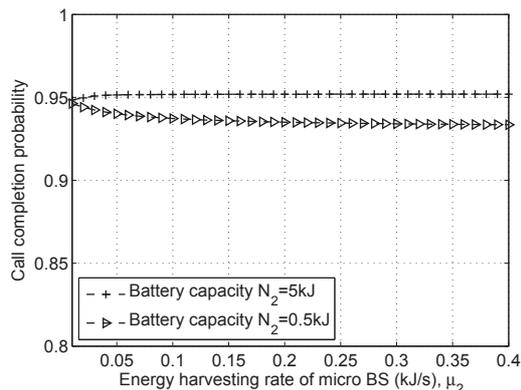
(b) Effect of energy harvesting rate for micro BSs.

Fig. 3. Call completion probability versus the energy harvesting rate, with different battery capacity.

C. Effect of Energy Harvesting Rate

Fig. 3 plots the call completion probability versus the energy harvesting rate for macro BS and micro BS, respectively, with different battery capacity. Fig. 3(a) shows that when the battery capacity of macro BSs is large, the call completion probability increases as μ_1 increases. However, when the battery capacity of macro BSs is small, the call completion probability first decreases and reaches a minimum, before increasing again. This finding be explained as follows. For the *macro BSs* when μ_1 increases, $\mathbb{E}[J_1^{ON}]$ in (14) increases, $\mathbb{E}[J_1^{OFF}]$ in (11) decreases and $\mathcal{P}_1^a$ in (15) increases. For the *micro BSs*, $\mathbb{E}[J_2^{ON}]$ in (10) depends on μ_1. This is because $v_2 = D_2\overline{A_2}\lambda_u$ and A_2 in (3) depends on both $\mathcal{P}_1^a$ in (15) and $\mathcal{P}_2^a$. Thus, when μ_1 increases, $\mathbb{E}[J_2^{ON}]$ and $\mathcal{P}_2^a$ slightly increase while $\mathbb{E}[J_2^{OFF}]$ is a constant. As a result, $\mathcal{P}_1^s$ increases and $\mathcal{P}_2^s$ decreases. The relative impact of these factors in (8) is influenced by the battery capacity of macro BSs. For small N_1 when μ_1 is also small, $\mathbb{E}[J_2^{ON}] \gg \mathbb{E}[J_1^{ON}]$ and is the dominant term. Hence, as μ_1 increases from a small value, the call completion probability slightly decreases. With increasing μ_1, once $\mathbb{E}[J_1^{ON}]$ exceeds $\mathbb{E}[J_2^{ON}]$, the call completion probability starts to increase with increasing μ_1. For large N_1 even when

μ_1 is small, $\mathbb{E}[J_2^{ON}] > \mathbb{E}[J_1^{ON}]$ but the magnitudes are comparable. Hence, the combined effect in (8) of the product of increasing $\mathcal{P}_1^s$ and increasing $\mathbb{E}[J_1^{ON}]$ is dominant over the effect of product of decreasing $\mathcal{P}_2^s$ and increasing $\mathbb{E}[J_2^{ON}]$. Hence the mean cell dwell time and the call completion probability increases as μ_1 increases.

Fig. 3(b) shows that the impact of increasing μ_2 is different to that of increasing μ_1. For sufficiently large micro BS battery capacity (equivalently, N_2^c is large), the call completion probability slightly increases as μ_2 increases. However, the call completion probability slightly decreases with increasing μ_2 when the battery capacity is small. This can be easily explained using similar arguments as before. Intuitively, when the micro BS battery capacity is small, micro BSs switch ON and OFF more frequently as the energy harvesting rate increases and this slightly negatively impacts call completion probability.

V. CONCLUSIONS

In this paper we have analysed the call completion probability of a two-tier heterogeneous network with energy harvesting BSs using a realistic BS energy consumption model. Using stochastic geometry, we have derived tight upper and lower bounds on the call completion probability in the presence of Rayleigh fading and interference, where the hand-off of a call is governed by BSs switching the ON and OFF. We have examined the impact of the system parameters (i.e., battery capacity, the minimum energy level at which BS switches back ON and energy harvesting rate) on the call completion probability. We have showed that the macro BS energy harvesting parameters have the dominant impact on the call completion probability.

APPENDIX A
PROOF OF LEMMA 1

Proof: Since the call duration T_c is exponentially distributed with parameter $1/M_c$, it has a probability density function (pdf) given by

$$f_{T_c}(t_c) = \frac{1}{M_c}e^{-\frac{t_c}{M_c}}. \qquad (16)$$

Following from (35) in [19], we have

$$\mathcal{P}^c = \int_0^\infty e^{-at_c - bt_c} f_{T_c}(t_c)dt_c$$
$$= \frac{1}{M_c}\int_0^\infty e^{-at_c - bt_c - \frac{t_c}{M_c}}dt_c$$
$$= -\frac{1}{M_c(a+b+\frac{1}{M_c})}\left[e^{-(a+b+\frac{1}{M_c})t_c}\right]_0^\infty$$
$$= \frac{1}{M_c(a+b)+1}, \qquad (17)$$

where M_c is the average call holding time, $a = \frac{\rho_f}{\rho_f\mathbb{E}[R]+(1-\rho_f)\mathbb{E}[T]}$, and $b = \frac{\rho_{link}}{\mathbb{E}[V]}$. Furthermore, as $0 \le \mathbb{E}[R] \le \mathbb{E}[T]$, which makes $\frac{\rho_f}{\mathbb{E}[T]} \le a \le \frac{\rho_f}{(1-\rho_f)\mathbb{E}[T]}$, $\mathcal{P}^c$ is then bounded by

$$\frac{1}{M_c(\frac{\rho_f}{(1-\rho_f)\mathbb{E}[T]}+b)+1} \le \mathcal{P}^c \le \frac{1}{M_c(\frac{\rho_f}{\mathbb{E}[T]}+b)+1}. \qquad (18)$$

As the hand-off is resulted from the BSs turning ON and OFF in the two-tier HetNet, the probability mass function of dwell time T is given by

$$\Pr(T = t) = \begin{cases} \mathcal{P}_1^{\mathrm{s}}, & t = J_1^{ON}; \\ \mathcal{P}_1^{\mathrm{s}}, & t = J_2^{ON}; \end{cases} \qquad (19)$$

where $\mathcal{P}_k^{\mathrm{s}}$ is the kth tier association probability.

We then can have the mean cell dwell time as shown in (8). Substituting (8) into (18), we arrive at (9). ■

APPENDIX B
PROOF OF LEMMA 2

Proof: The static energy consumption of a BS does not play a role in the birth and death process. During the expected ON time of the micro BS, the energy consumption contributed by the static part is $S_k \mathbb{E}[J_k^{ON}]$. Using superposition, the expected ON time of a BS is equivalent to the mean hitting time from state $N_k^{\mathrm{c}} - S_k \mathbb{E}[J_k^{ON}]$ to 0, where the mean hitting time is defined as the expected time that the state reaches 0 for the first time starting from $N_k^{\mathrm{c}} - S_k \mathbb{E}[J_k^{ON}]$.

For the kth tier BS, the generator matrix of a birth and death process with constant birth and death rate is defined as [21]

$$\vec{G} = \begin{bmatrix} -\mu_k & \mu_k & 0 & \ldots & 0 & 0 \\ v_k & -v_k - \mu_k & \mu_k & \ldots & 0 & 0 \\ 0 & v_k & -v_k - \mu_k & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & -v_k - \mu_k & \mu_k \\ 0 & 0 & 0 & \ldots & v_k & -v_k \end{bmatrix} \qquad (20)$$

Using the analysis technique in [14], the mean hitting time is

$$\mathbb{E}[J_k^{ON}(N_k^{\mathrm{c}})] = \left((-\mathcal{B}_k)^{-1} \mathbf{I} \right) (N_k^{\mathrm{c}}), \qquad (21)$$

where $\mathbf{I}$ is a column vector of all 1s and $\mathcal{B}_k$ is a $(N_k - 1) \times (N_k - 1)$ sub-matrix of $\vec{G}$ without its first column and first row. Using the results in [14], the (i, j)th element of $(-\mathcal{B}_k)^{-1}$ can be expressed as

$$(-\mathcal{B}_k)^{-1}(i, j) = \frac{-1}{v_k^j} \sum_{n=1}^{\min(i,j)} \mu_k^{j-n} v_k^{n-1}. \qquad (22)$$

Therefore the expected ON time can be derived as

$$\mathbb{E}[J_k^{ON}] = \sum_{j=1}^{N_k} \frac{1}{v_k^j} \sum_{n=1}^{\min(i,j)} \mu_k^{j-n} v_k^{n-1}$$

$$= \frac{\left(\frac{\mu_k}{v_k} \right)^{N_k} - \left(\frac{\mu_k}{v_k} \right)^{N_k - N_k^{\mathrm{c}} + S_k \mathbb{E}[J_k^{ON}]}}{(v_k - \mu_k)^2 \mu_k^{-1}} - \frac{N_k^{\mathrm{c}} - S_k \mathbb{E}[J_k^{ON}]}{\mu_k - v_k}. \qquad (23)$$

■

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