An Efficient Energy Curtailment Scheme For Outage Management in Smart Grid

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***Abstract*—In this paper an efficient energy curtailment scheme is studied, which enables the power users of a smart grid network to decide on the reduction in energy supplied to them in the event of a power outage in the system. Considering the advantages of a two-way communications infrastructure for any future smart grid, a non-cooperative generalized Nash game is proposed where the players are users of power in the network. They adopt a strategy to choose the amount of reduction in energy supplied to them based on their energy requirements so as to minimize the total cost incurred to the system due to the power outage (i.e., social optimality). The game is modeled as a variational inequality problem, and it is shown that the socially optimum solution is obtained at the variational equilibrium of the energy curtailment game. An algorithm that enables the users to efficiently reach this equilibrium is proposed. Simulation results show that the proposed game yields an improvement of about 15% on average, in terms of average total cost reduction, compared to a standard equal power curtailment scheme.**

***Index Terms*—Smart grid, energy curtailment, outage management, game theory, variational inequality, variational equilibrium.**

I. Introduction

Increased expectations of customers, limited energy resources and the expensive process of exploiting new resources have put the reliability of the power grid in danger [1]. Especially in the event of a power outage in the grid system, lack of "situational awareness" is often the main reason that leads to a large scale fault event which may causes an extensive cost to the whole system [2]. For instance, the annual cost of power outage in United States in 2002 was estimated to be on the order of \$79 billion [3], rising to \$100 billion in 2007 [4]. Therefore, the study of an efficient outage management scheme in the event of a power disruption in the grid (e.g., a scheme for optimal curtailment of electricity from the customers) to reduce its catastrophic impact on the whole grid system is of paramount importance to improve the system's reliability.

It is envisioned that a smart grid will transform the current power grid into one that functions more intelligently, giving better situational awareness and providing resilience against component failures and disastrous impacts on the grid [3]. In a smart grid network energy consumers are able to actively take part in the decision making process regarding various grid management issues, and then agree on parameters that better protect the grid from any undesirable fault event [5]. Hence in recent years, extensive research has been devoted to system reliability and pre and post-failure protection of the smart grid, e.g., [2, 6–10].

One of the key challenges for reliable smart grid operation is the post-outage management of power among the users immediately after a power disruption in the network. A power outage may occur due to a fault in the power line or due to the intermittent nature of renewable energy sources [5]. Hence, there is a need of curtailment of power from the users in the network so as to manage the activities of the grid effectively during the outage time until the whole system can operate normally again. Of foremost importance is efficient curtailment of energy so there is minimum total cost to the whole system. It is important to note that even a reduction in the total cost by 1% can significantly benefit the users in the system. For example, the annual cost of outage in the U.S.A. in 2002 would be reduced by up to \$790 million with only 1% reduction in total cost of outage [3]. Thus, there is a need to develop solutions that will be able to optimally reduce the total cost incurred to a system during periods of power outage, and thereby to benefit the power system users.

The main contribution of this paper is to model an efficient outage management scheme for smart grid taking the advantage of its two way communications infrastructure. An efficient energy curtailment scheme for cutting off energy from the users in the event of a power outage is proposed to minimize the total cost incurred to the system for this outage. We study an energy curtailment game (ECG) where the users play a generalized Nash game amongst themselves to decide on the amount of energy to be curtailed from them in the event of a power disruption. We investigate the properties of the game and show that there exists a socially optimal solution. The socially optimal solution is where the total cost incurred reaches a global minimum. An algorithm is proposed to obtain this optimal solution. Through simulation, the performance of

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the game is assessed and the effectiveness of the scheme is demonstrated by comparing it with a standard equal energy curtailment scheme.

The rest of the paper is organized as follows: the system model is presented in Section II. A non-cooperative generalized Nash game is formulated and its properties are studied in Section III. In Section IV, the solution method of the game is discussed and an algorithm is proposed to reach the equilibrium. Numerical results for the proposed scheme are given in Section V. Finally, Conclusions are drawn in Section VI.

II. SYSTEM MODEL

Consider a smart grid system consisting of a single energy source (ES) and multiple energy users (EUs). The ES can be a single energy generating unit or an aggregation of multiple distributed renewable energy generating units in the network such as wind farms, smart homes, solar farms, bio-gas plants and plug-in hybrid electric vehicles (PHEVs) acting as a single virtual power plant in the system. It is assumed that each EU is equipped with a smart meter that has a decision making capability on the amount of energy to be curtailed and each EU is also connected to the ES by means of a power line [11]. The smart meters are also connected to the ES through a local area network (LAN). All communications between the ES and EUs take place using an appropriate communication protocol (e.g., Zigbee [12]).

Throughout the paper, it is assumed that $\mathcal{N}$ denotes the set of EUs in the network where the number of EUs is $N = |\mathcal{N}|$. Each EU $n \in \mathcal{N}$ is a single user in the network, for instance a smart home. Renewable energy generation is subjected to wide fluctuations and the available energy for the consumers may vary significantly with time [13]. In the event of an unfavorable circumstance, e.g., a cloudy day making solar energy generation unproductive, or the failure of a few energy generating units, a power disruption may occur in the smart grid system. Therefore, the ES would be unable to meet the total energy demand of its customers in the network for a particular period of time, e.g., before restoration of full service. Let us assume that for a particular duration of time of the day E_d is the total energy demand of the consumers and E_a is the available energy to the ES at the event of power outage. If $E_d > E_a$, the ES will be unable to meet the excess demand,

$$E_x = E_d - E_a, \tag{1}$$

of the power users in the network for that period of time. So this amount of energy must be curtailed from the EUs and the users will experience a black-out. If e_n is the amount of energy which is to be curtailed from user n, the curtailment of energy e_n from each user n needs to maintain the constraint

$$\sum_n e_n = E_x. \tag{2}$$

According to constraint (2), the total deficiency of energy E_x is overcome by suitable curtailing of energy from all users and is necessary for reliable power distribution in the smart grid network.

Energy requirements of EUs may vary based on different factors, such as the time of the day or the type of EU. For example, a school requires less energy, and maintaining full energy supply is less important, during vacation than during term time. Hence, such factors must be taken into account when designing an energy curtailment scheme for the EUs. Thus, the main challenges faced during the decision making process for energy curtailment in a smart grid in the event of a power outage are:

1) modeling the decision making process of energy curtailment from the energy users given the constraint in (2);
2) capturing the EUs' requirements for energy during the decision making process for curtailment; and
3) developing an algorithm that enables the EUs to optimize the amount of energy to be curtailed from them so as to minimize the total cost incurred to the system.

To address the above challenges, first we define a cost function for each EU in Section II-A and then formulate the decision making process as a constrained optimization problem in Section II-B.

A. EU's cost function

To capture the effects of energy outage on the overall smart grid system, we define a cost function c_n for each EU $n \in \mathcal{N}$ in the network, which represents the cost incurred by the EU n due to the curtailment of e_n from it. The choice of cost function is based on a linearly decreasing cost with decrease in energy supply, which has recently been shown to be appropriate for users of power [1]. The cost function c_n for EU n is defined as, [14],

$$c_n(e_n, \theta_n) = k_1 e_n^2 + k_2(e_n - \theta_n e_n), \tag{3}$$

where $k_1, k_2 > 0$ are the scaling factors and θ_n is the customer preference parameter (CPP) of EU n [14]. The CPP, θ_n, is a measure of each EU's preference for the amount of energy to be curtailed from it. For example, curtailment of 1 kWh of energy may have far worse impact on industry than on a residential home. Thus, the CPP could be very different for a residential home than for an industry for the same energy curtailment. As from (3), a higher CPP leads to a lower cost for an EU and hence, an EU with higher CPP would be able to endure the cost of more curtailment of energy than the EU with lower CPP. The cost function in (3) is assumed to possess the following properties:

- the cost for EU n increases as the amount of energy to be curtailed from it increases. That is

$$\frac{\delta c_n}{\delta e_n} > 0. \tag{4}$$

- an EU with higher CPP will experience less cost compared to an EU with lower CPP for the same energy curtailment. Therefore,

$$c_n(e_n, \widetilde{\theta}_n) < c_n(e_n, \widehat{\theta}_n), \ \forall \ \widetilde{\theta}_n > \widehat{\theta}_n. \tag{5}$$

B. Problem formulation

It is clear from (3) that, for a fixed amount of energy curtailed from an EU, the cost incurred by the EU changes as the CPP of the EU changes. Hence, each EU n in the smart grid network needs to optimally choose an amount of energy e_n to be curtailed from it so as to minimize the overall cost incurred to the whole system. Thus, the objective of each EU is

$$\begin{aligned}\min_{e_n}\sum_n c_n(e_n,\theta_n) &= \min_{e_n}\sum_n \left(k_1e_n^2 + k_2(e_n-\theta_n e_n)\right),\\ \text{subject to } \sum_n e_n &= E_x,\ \forall n\in\mathcal{N}. \end{aligned} \tag{6}$$

The optimization problem in (6) can be solved by the ES in a centralized fashion if the ES knows all the parameters. However, in smart grid the ES may not have full control over the decision making process of the EUs [15], and in particular the ES may not know the preference parameter θ_n when EUs keep θ_n as confidential. Therefore, a decentralized decision making scheme is required for the EUs to voluntarily choose an amount of energy to be curtailed from them for the whole system's social benefit. Next we will show that the optimization can be achieved with limited coordination by the ES and without letting the ES know the e_n that each EU contributes.

III. Generalized Nash game

To study the energy curtailment scheme for outage management in a smart grid, we use the framework of a generalized Nash game [16]. A generalized Nash game (GNG) is a type of game that allows joint constraints for all players involved in the game [16]. In the proposed game, the players are the energy users in the network, which choose the amount of energy to be curtailed from them subject to the joint constraint in (2). The game is formally defined by its strategic form

$$\chi = \{\mathcal{N}, \mathbf{E_n}, c\}, \tag{7}$$

which has the following components:

(i) the set of energy users in the smart grid network $\mathcal{N}$.
(ii) the strategy vector $\mathbf{E_n}$ of each player $n\in\mathcal{N}$, which refers to the amount of energy to be curtailed $e_n\in\mathbf{E_n}$ from n satisfying the constraint $\sum_n e_n = E_x$.
(iii) the total cost incurred by all the EUs c due to the curtailment of energy e_n from the EU n for all $n\in\mathcal{N}$.

It is important to note that the action of each EU n, to optimize (6), affects the choice of actions of other EUs in the network due to the presence of (2). Thus, the proposed GNG is a jointly convex generalized Nash equilibrium problem (GNEP) with coupled constraint (2) [16]. To solve this jointly convex GNEP, we formulate the game as a variational inequality problem (VIP) and investigate the existence of the variational equilibrium (VE) of the game [17]. The VE is the socially optimal outcome of a GNEP and a valuable target solution for the proposed energy curtailment scheme. This is due to the fact that a socially optimal solution leads to the minimum total cost for the whole system. Thus the socially optimal solution is the main target of the proposed game. In the following we investigate the existence of a socially optimal solution of the proposed GNG.

A. Existence of a socially optimal solution

A pure strategy solution in a non-cooperative game is not always guaranteed [18]. Therefore, we investigate the existence of a solution of the proposed GNG and study its optimality, if it exists. Now, the joint cost function in (6) of the proposed game can be expressed as, [19],

$$c(\mathbf{e}) = \sum_{n=1}^{N} c_n(e_n,\theta_n), \tag{8}$$

where

$$\mathbf{e} = \begin{bmatrix} e_1\\ e_2\\ \vdots\\ e_N\end{bmatrix} \in \mathbf{E} \tag{9}$$

and $\mathbf{E}$ is included in the definition of joint convexity [16]. Due to the jointly convex nature of the GNEP, the proposed GNG can be formulated as a variational inequality problem VIP(**E**,**F**) [16] where $\mathbf{F} = (\nabla_e c_n(\mathbf{e}))_{n=1}^N$ is the pseudo-gradient of (8). Now, to show the existence of an optimal solution, we will prove the following theorem:

Theorem 1: A variational equilibrium (VE) exists for the proposed VIP(**E**,**F**) and the VE is unique.

Proof: To prove this, we need only to prove that the pseudo-gradient of $c(\mathbf{e})$ monotonically increases with e_n for fixed k_1, k_2 and θ_n [19]. The pseudo-gradient of $c(\mathbf{e})$ is

$$\mathbf{F} = \begin{bmatrix} 2k_1e_1 + k_2(1-\theta_1)\\ 2k_1e_2 + k_2(1-\theta_2)\\ \vdots\\ 2k_1e_N + k_2(1-\theta_N)\end{bmatrix} \tag{10}$$

and the Jacobian of $\mathbf{F}$ is [17],

$$\mathbf{JF} = \begin{bmatrix} 2k_1 & 0 & \dots & 0\\ 0 & 2k_1 & \dots & 0\\ \vdots & \vdots & \dots & \vdots\\ 0 & 0 & \dots & 2k_1\end{bmatrix}. \tag{11}$$

In (11), the scaling factor k_1 is always positive. Now considering the i^{th} leading principal minor (LPM) $\mathbf{JF}_i$ of the leading principal sub-matrix[1] $\mathbf{JF}$, it can be shown that $\mathbf{JF}_i$ is always positive (i.e., $|\mathbf{JF}_1| > 0, |\mathbf{JF}_2| > 0$, and so on). Hence, $\mathbf{JF}$ is positive definite on $\mathbf{E}$, and thus, $\mathbf{F}$ is strictly monotone. So, VIP(**E**,**F**) admits a unique VE solution [16]. ■

Remark 1: An important result of Theorem 1 is that the VE is the socially optimal solution of the proposed GNG. This can be explained as follows: from Theorem 1, the VE is unique for the proposed VIP($\mathbf{E},\mathbf{F}$). On the other hand, the proposed

[1]The i^{th} order principal submatrix $\mathbf{A}_i$ can be created by deleting the last $g-i$ rows and last $g-i$ columns from $g\times g$ matrix $\mathbf{A}$.

Algorithm 1 Algorithm to reach VE

1. The ES announces E_x to the EUs in the network.
2. Each EU n estimates an amount e_n to be curtailed using the S-S method [20]:
   S-S method
   a) At iteration k, EU $n \in \mathcal{N}$ computes the hyperplane projection $r(e_n^k)$ and updates $e_n^{k+1} = r(e_n^k)$.
   b) The EU checks: if $r(e_n^k) = 0$
   if $r(e_n^k) = 0$
   a) The EU chooses the energy e_n to submit to the ES.
   else
   a) the EU n determines the hyperplane z_n^k and the half space H_n^k from the projection.
   b) the EU updates the amount e_n^{k+1} from the projection of its previous energy e_n^k on to $X \cap H_n^k$ and choose to submit to the ES.
   end if
3. The EU n calculates λ_n using Proposition 1 and submits it to the ES.
4. The ES checks $\lambda_n, \ \forall n \in \mathcal{N}$.
   if $\lambda_1 = \lambda_2 = ... = \lambda_N = \lambda$
   The ES determines the VE energy vector $\mathbf{e}^*$ of all the EUs in the network.
   else
   The ES directs the EUs to Repeat step - 2 and Step - 3.
   end if

The VE energy choices of all the energy users in the network $\mathbf{e}^*$ are obtained.

GNEP has already been shown to be a jointly convex GNEP. Hence, the VE is the unique global minimizer of (8) [16]. Thus, in other words, it is also the socially optimal solution of the proposed GNG. We will use VE to indicate the solution of the proposed GNG for the rest of the paper.

IV. Game solution and algorithm

Each EU plays a GNG, to choose an amount of energy to be curtailed, by solving the variational inequality problem. Each EU n wants to minimize the total cost incurred to the system by suitably choosing its strategy e_n subject to (2). The total cost is minimized as soon as the solution of the game reaches the VE.

Definition 1: Consider the GNG χ given in Section III where the joint cost function c is defined as in (8). A vector of strategies $\mathbf{e}^*$ constitutes the VE of the game if and only if it satisfies the following set of inequalities:

$$c(\mathbf{e}^*) \leq c(\mathbf{e}), \ \forall e_n \in \mathbf{E_n}, \ n \in \mathcal{N}, \tag{12}$$

where $\mathbf{e}^* = [e_1^*, e_2^*, ..., e_N^*]^T$ and $\mathbf{e} = [e_1^*, e_2^*, ..., e_{n-1}^*, e_n, e_{n+1}^*, .., e_N^*]$ for one or more $n \in \mathcal{N}$. Thus, the VE defines a state of the game in which the total cost incurred by the system cannot be reduced if any EU deviates from its VE strategy and chooses a different amount of energy to be curtailed from it, given other EUs are playing their VE strategies.

To find the VE of the game the EUs in the network have to solve a VIP. As shown in Section III, the VIP of the proposed scheme is strictly monotonic and thus, the EUs can reach the VE of the game by solving a monotone VIP. To find the solution of the proposed monotone VIP, it will be useful to state the following proposition which characterizes the solution of a strictly monotone VIP [16].

Proposition 1: For a strongly monotone VIP, the slack variable $\lambda_n = \delta c_n / \delta e_n$ possesses the same value λ at the VE solution for all the EUs in the smart grid network. That is, $\lambda_1 = \lambda_2 = ... = \lambda_N = \lambda, \ \forall n \in \mathcal{N}$.

This property is used to determine the VE of the proposed GNG. We use a hyperplane projection method[2], particularly the iterative S-S method [20], to solve the proposed monotone variational inequality problem to determine the EUs' decisions on the energy vector. The algorithm, as detailed in Algorithm 1, requires limited communication between the EUs and the ES in the network[3]. The algorithm starts with an announcement by the ES of the total energy deficiency in the network and the EUs play a GNG amongst themselves to reach an optimal solution. Each EU n solves the S-S hyperplane projection method to choose its energy curtailment and calculates λ_n from its choice using Proposition 1. The EUs submit these $\lambda_n, \ \forall n \in \mathcal{N}$, to the ES. The ES checks λ_n for each EU n and informs the EUs as to whether all λ_n's are equal. The algorithm converges to the optimal VE and the curtailment of energy takes place as soon as λ_n converges to the single value $\lambda_n = \lambda, \ \forall n \in \mathcal{N}$. For a strongly monotone VIP, the hyperplane projection method is guaranteed to converge to a non-empty solution [21] and thus, the proposed algorithm always converges to a non-empty optimal solution.

V. Numerical Simulation

To simulate the proposed energy curtailment scheme, we consider a number of EUs in the network and simulate the scheme for different scenarios. The consumer preference parameter is chosen from a uniform random distribution between 0 and 1 [14]. The value of k_1 and k_2 is chosen in such a way that $\frac{k_1}{k_2} = 0.5$ is maintained [14]. The minimum deficiency in energy is chosen as 2 kWh and the maximum deficiency is chosen as 10 kWh for the duration of the power outage[4].

To show the convergence of the proposed algorithm to the VE, we assume a network with five EUs in which they are playing a generalized Nash game amongst themselves to reduce the effect of a total energy outage of an amount of 10 kWh on the system. In Fig. 1 and Fig. 2, we show the convergence of the amount of energy that has to be curtailed from each EU to the VE and also the convergence of the cost due to this curtailment. From Fig. 1 and Fig. 2 it can be seen that as the amount of energy to be curtailed reduces, the cost incurred to the corresponding EU decreases and vice versa. This is due to the variation in the customer preference parameter, CPP, or each EU, as the choice of energy curtailment is highly dependent on the CPP. Hence, by choosing more curtailment by the EU with higher CPP and lower curtailment by the EU with lower CPP, the total cost incurred by the system can be reduced to a minimum at the VE. Both the choice of energy and the cost incurred by

[2]Projection $r(z) = \arg\min\{||w - z||, w \in \mathbf{E}\}, \ \forall z \in \mathcal{R}^n$.

[3]For instance, a single bit can be sent by the ES to the EU n if the EU's strategy does not satisfy (2) given the strategies of other EUs.

[4]Clearly this energy deficiency range is highly variable and could be different for many scenarios.

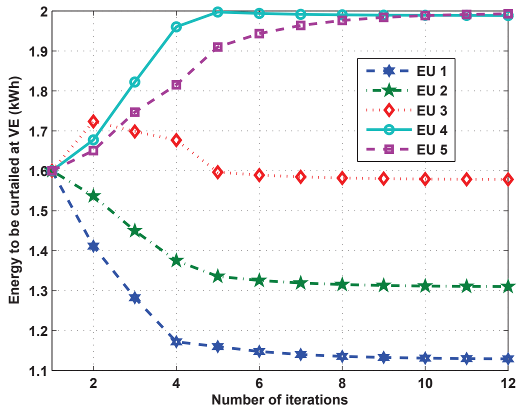

Fig. 1: Convergence of the amount of energy to be curtailed from each EU to the variational equilibrium.

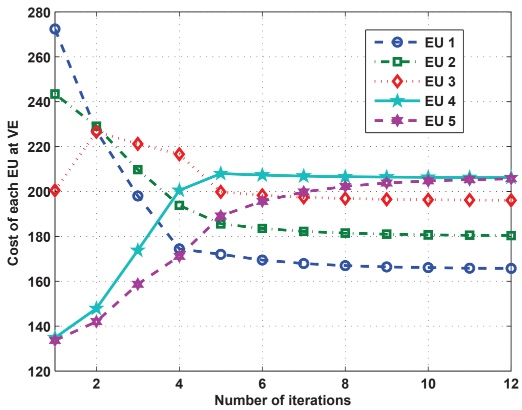

Fig. 2: Convergence of the cost incurred by each EU to the variational equilibrium.

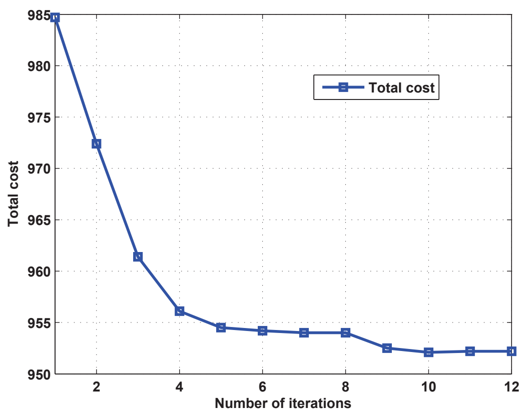

Fig. 3: Convergence of the total cost of the system to the variational equilibrium.

each EU converges to the VE after the 10^{th} iteration of the algorithm.

The total cost incurred by the system at VE is shown in Fig. 3, where we see that the total cost is reduced to its minimum value after 10 iterations. Although the cost, incurred at variational equilibrium, to each EU does not achieve a minimum value for all EUs, as seen in Fig. 2, the total cost to the system is minimized at equilibrium. This is due to a preference based choice of energy by the EUs in the network. An EU with higher CPP can endure the impact of more energy curtailment (i.e., more costs) compared to the other EUs with lower CPP. Thus, higher cost to an EU with higher CPP does not affect the system, due to its higher tolerance to reductions in energy supply.

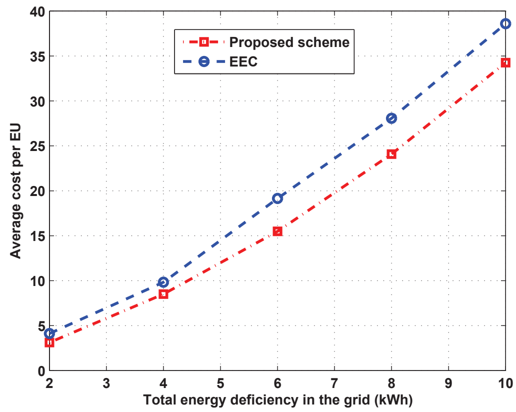

Fig. 4: Effect of the total energy deficiency on the cost of each EU.

The performance benefit of the proposed scheme is assessed in Fig. 4 and Fig. 5. The effect of total energy deficiency on the average cost incurred by any EU in the network is shown in Fig. 4. The effect of the number of EUs on the total average cost of the system is shown in Fig. 5. In both Fig. 4 and Fig. 5 we compare our proposed scheme with a standard equal energy curtailment (EEC) scheme [22] in which an equal amount of energy is curtailed from each of the users in the network to mitigate the effect of total system energy deficiency.

In Fig. 4, for a fixed number of EUs in the network, the effect of variation in total system energy deficiency on the average cost per EU is observed. It is shown for both the EEC and the proposed case that the average cost per EU increases as the system's total energy deficiency increases. Because of more energy curtailment allowed by the EUs the average cost incurred per EU increases. However, the average cost per EU is less for our scheme than for the EEC scheme. As shown in the figure, the average costs per EU for the proposed case are 0.76, 0.86, 0.78, 0.85 and 0.88 times the cost of the EEC scheme for energy deficiencies of 2, 4, 6, 8 and 10 kWh respectively. Therefore, there is an average reduction in cost of 17% for our scheme. The CPP of each EU enables it to decide on the amount to be curtailed to reduce the total cost to the system and thus obtain performance benefit in terms of average cost per EU relative to the EEC scheme.

The effect of the number of EUs on the average total cost for the system is shown in Fig. 5. Assuming a total 10 kWh

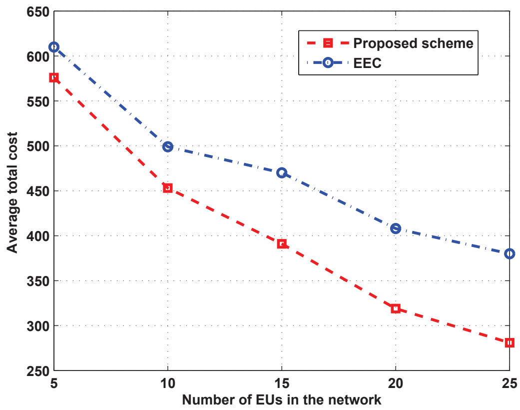

Fig. 5: Effect of the number of EUs on the total cost of the system.

energy deficiency, we increase the number of EUs in the network from 5 to 25 and observe impact on total cost for both the proposed scheme and the EEC scheme. In Fig. 5 the total cost for the system decreases as the number of EUs in the network increases. The reason behind this phenomenon is that as the number of EUs in the network increases less energy can be curtailed from each EU to address the total system energy deficiency, and consequently the average total cost decreases. It is also observed in Fig. 5 that the average total cost for our proposed scheme is between 5.6% and 26% less than the EEC scheme, and on average 15% less than the EEC scheme, with increasing performance benefit for our scheme as the number of EUs increases. The performance benefit is due to the fact that in the proposed scheme the EUs decide on their energy curtailment amount based on their preferences, and optimally choose the amount to be curtailed from them. On the other hand, in the EEC scheme the energy is curtailed uniformly from all the users in the network. Hence, this optimal selection of the energy leads to an improvement for the proposed scheme in terms of total cost incurred to the smart grid system compared to the EEC scheme. Furthermore, such improvement in total cost becomes more pronounced as the number of EUs increases.

VI. Conclusion

In this paper we have introduced an approach for outage management in smart grid to address power disruption in the system. We have formulated a non-cooperative generalized Nash game among the energy users in the network. In the game each user strategically chooses an amount of energy to be curtailed from them based on their preference parameter. We have studied the properties of the game and showed that the game leads to an optimal solution for curtailment. By formulating the game as a variational inequality problem we have proposed an algorithm that enables the users of energy in the network to choose their strategies of energy curtailment and reach the optimal solution for the game. With simulations, it has been shown that the total cost to the smart grid system due to this energy outage converges to a minimum at the variational equilibrium. We have compared our scheme with an equal energy curtailment scheme and have demonstrated improvement in terms of reduction in average total cost and reduction in average cost per user in the system. Considering the proposed scheme, potential future extensions of this work include determining computational complexity, for any number of EUs, and determining optimal network sizes.

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