Effect of Vehicle Mobility on Connectivity of Vehicular Ad Hoc Networks

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Abstract— **Connectivity is a fundamental requirement in the planning, design and evaluation of vehicular ad hoc networks (VANET). In this paper, we propose a new equivalent speed parameter and develop an analytical model to explain the effect of vehicle mobility on the connectivity of highway segments in a VANET. We prove that the equivalent speed is different from the average vehicle speed and it decreases as the standard deviation of the vehicle speed increases. Using the equivalent speed we derive a novel analytical expression for the average number of vehicles on a highway segment, which allows us to accurately predict the network 1-connectivity. We verify the correctness of our analytical approach by comparing the numerical results with simulations. The results show that increasing the average vehicle speed increases the equivalent speed, which leads to a decrease in the average number of vehicles on a highway segment and consequently degrades connectivity. On the other hand increasing the standard deviation of the vehicle speed decreases the equivalent speed, which leads to an increase in the average number of vehicles on a highway segment and consequently improves connectivity. The results also show that vehicles in a VANET can adaptively choose their transmission range to ensure network connectivity in highway segments while minimising power consumption.**

I. Introduction

Vehicular ad hoc networks (VANETS) are an important emerging application of mobile ad hoc network [1]. Currently the IEEE 1609 Wireless Access in Vehicular Environments (WAVE) trial use standards, which use IEEE 801.11p, are being developed for VANET applications [2]. The potential applications of VANETS include safety-related applications such as cooperative forward collision warning system, traffic signal violation warning, lane change warning and information/entertainment applications for back seat passengers [1]. All these applications rely on successful dissemination of time-critical information to all vehicles on a road segment. Hence network connectivity is a fundamental requirement in VANETs, i.e. all vehicles on a road segment should be able to communicate with each other either directly or via multiple hops between intermediate vehicles. In a VANET the scenario of sufficiently high vehicle density, e.g. at busy intersections, corresponds to a trivial case from connectivity view point as the network will be almost surely. Therefore, characterizing connectivity of VANETs when the vehicle density is low and the speed of the vehicles and the flow are independent, i.e. in free-flow phase, is a crucial research challenge for realizing commercial VANET applications.

The study of VANET connectivity in free-flow phase has attracted considerable attention recently [3]–[13]. Assuming stationary nodes distributed along a straight line, the exact probability of connectivity of a one-dimensional ad hoc network is derived in [3] using the concept of random division of an interval and in [4] using a queueing theory approach. Note that the above two results are not directly applicable to VANETs because of the assumption of stationary nodes. The connectivity in sparse one-way vehicular ad hoc networks using a simple car following model, i.e. assuming all vehicles maintain a constant speed while on the highway and no overtaking is allowed, is examined in [5]. The results are extended to the case of two-way highway scenarios in [6] and multihop connectivity in [7]. More realistic mobility models, assuming normally distributed vehicle speeds and allowing vehicles to freely pass each other, are proposed in [8] and [9] and connectivity results derived using different approaches. A comprehensive model of vehicle mobility for determining VANET connectivity, which considers nodes arriving and departing at predefined entry and exit points along a highway, is presented in [10]. A vehicle mobility model taking into account the topology of roads and two-dimensional movement of vehicles along streets is proposed in [11] and connectivity bounds derived for sparse and dense VANETs. Finally, connectivity results using commercial car traffic simulators are reported in [12], [13]. A major limitation of all the above mentioned works is that while the connectivity results show that higher mobility generally degrades the network connectivity, the *exact dependance* on the mean and standard deviation of vehicle speeds is not addressed analytically.

In this paper, we propose a new equivalent speed parameter to accurately characterise the effect of vehicle mobility on connectivity of VANETs. Our novel approach to characterize the vehicle mobility is based on viewing a single stream of vehicles with random distributed speeds as a sum of infinite number of parallel virtual streams, where each virtual stream is comprised of vehicles with a particular constant speed. Using this mobility model, we study the connectivity of vehicular ad hoc networks in free-flow state from the point of view of connectivity of individual highway segments. This is important because it allows us to identify the bottleneck segments with low connectivity. The major contributions of this paper, in comparison to previous research, are as follows:-

- We show that the equivalent speed μ_{eff} is different from the average vehicle speed, μ, and decreases as the standard deviation of the vehicle speed, σ, increases. Our

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work differs from [9], where it is proposed to use the average speed (given a distribution of vehicle speed) as the equivalent speed.

- Using the equivalent speed, we derive an analytical expression for the average number of vehicles on a highway segment. This allows us to adapt the result in [3] to accurately predict the connectivity of a highway segment.
- We characterise the highway connectivity in terms of the connectivity of individual highway segments. The results can be used by vehicles at the start of a highway segment to adapt their transmission range to achieve desired network connectivity while minimising power consumption.

This paper is organised as follows. The system model is summarized in Section II. The derivation of the equivalent constant speed and the average number of vehicles, and the analytical model for highway segment connectivity is presented in Section III. The results are presented in Section IV. Finally, conclusions are drawn in Section V.

II. System Model

Consider a unidirectional, single-lane highway of length H meters. The transmission range of all vehicles is R meters, i.e. two vehicles can directly communicate if the distance between them is less than or equal to R m [1].

A. Highway Model

We assume that there are S service points located at distances d_s from the start of the highway, where $s = 1, 2, \ldots, S-1$ is the highway segment index. The first service point coincides with the start of the highway (i.e. $d_1 = 0$ m) and the last service point coincides with the end of the highway (i.e. $d_S = H$ m). The service points divide the highway into $S-1$ segments each of length $L_s = (d_{s+1} - d_s)$ m. Empirical studies have shown that Poisson distribution provides an excellent model for vehicle arrival rate in free-flow phase [5]. We, therefore, assume that the vehicles can arrive at the highway through any of the service points according to a Poisson distribution. The vehicles then move along the highway according to the assumed mobility model independent of all other vehicles. The vehicles can depart the highway through any service point. The end of the highway serves as the final exit point of all the vehicles. Note that a vehicle cannot depart from the service point at which it has just arrived and vehicles cannot arrive at the last service point which is located at the end of the highway.

Let x_{kj} denote the stream of vehicles with arrivals and departures at service points k and j, respectively ($j > k$). Then the arrival rate λ_{kj} of the x_{kj}th stream is given by [10]

$$\tilde{\lambda}_{kj} = \begin{cases} \lambda_k(1-\alpha_j) \prod\limits_{l=k+1}^{j-1} \alpha_l & , j \geq k+2 \\ \lambda_k(1-\alpha_j) & , j = k+1 \end{cases} \quad (1)$$

[1]Note that since $R \gg 3.6$ m which is the standard highway's lane width, a single lane highway model can also accurately describe multilane highways.

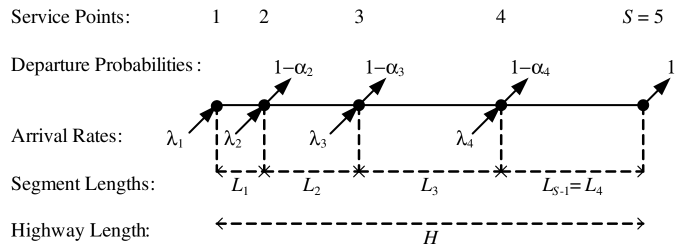

Fig. 1. Highway model (for simplicity, the case of $S = 5$ service points is shown).

where λ_k = arrival rate of vehicles at the kth service point, α_k = probability that old vehicles arriving from the preceding highway segment continue to the kth segment, $1 - \alpha_k$ = probability old vehicles arriving from the preceding highway segment depart from the highway at the kth service point. Note that for the last service point, from the highway end point constraints, we have $\alpha_S = 0$ (hence $1 - \alpha_S = 1$) and $\lambda_S = 0$. For simplicity, the highway model is illustrated in Fig. 1 for the case of $S = 5$ service points on a highway.

B. Mobility Model

Consider the scenario that each vehicle enters a highway segment with a random speed. For simplicity, we assume that the random vehicle speeds are independent and identically distributed and independent of vehicle arrival time. In general, each vehicle's speed can vary with time as the vehicle travels along the highway. It has been shown in [9] that for the case of vehicle speeds varying over time according to a wide sense stationary random process, the steady-state distribution of vehicle locations is the same as in the case where the vehicle speeds are constant over time. Further, empirical studies have shown that the speeds of different vehicles in free-flow state follow a Gaussian distribution [5].

We, therefore, assume that each vehicle is assigned a random speed v chosen from a Gaussian distribution and that each vehicle maintains its randomly assigned speed while it is on the highway. Note that overtaking is allowed in this mobility model. To avoid dealing with negative speeds or speeds close to zero, we use a truncated Gaussian Probability Density Function (PDF), given by

$$\hat{f}_V(v) = \frac{2 f_V(v)}{\text{erf}\left(\frac{V_{\text{max}} - \mu}{\sigma\sqrt{2}}\right) - \text{erf}\left(\frac{V_{\text{min}} - \mu}{\sigma\sqrt{2}}\right)} \quad (2)$$

where $f_V(v) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(v-\mu)^2}{2\sigma^2}\right)$ is the Gaussian PDF, μ = average vehicle speed, σ = standard deviation of the vehicles speed, $V_{\text{max}} = \mu + 3\sigma$ is the maximum speed, $V_{\text{min}} = \mu - 3\sigma$ is the minimum speed [8]. Typical values for μ and σ, derived from measurements, are given in Table I [14].

III. Analytical Model

In this section, we derive an analytical expression for the average number of vehicles in a highway segment and present an analytical model to determine the highway connectivity in steady-state free-flow phase.

TABLE I

TYPICAL VALUES FOR THE AVERAGE AND STANDARD DEVIATION OF THE VEHICLE SPEEDS ON A HIGHWAY [14].

μ (km/hr)	σ (km/hr)
70	21
90	27
110	33
130	39
150	45

A. Average number of vehicles

Theorem 1: Consider a highway segment of length L carrying a single stream of vehicles with arrival rate λ. The average number of vehicles N on the highway segment, under steady state, is given by

$$N = \frac{L\lambda}{\mu_{\text{eff}}} \tag{3}$$

where μ_{eff} is an equivalent speed which is given by

$$\mu_{\text{eff}} = \frac{1}{\frac{\frac{2}{\sqrt{2\pi}\sigma}}{\text{erf}\left(\frac{V_{\max}-\mu}{\sigma\sqrt{2}}\right)-\text{erf}\left(\frac{V_{\min}-\mu}{\sigma\sqrt{2}}\right)} \int\limits_{V_{\min}}^{V_{\max}} \frac{1}{v}\exp\left(-\frac{(v-\mu)^2}{2\sigma^2}\right)dv} \tag{4}$$

where all the parameters used in (4) have been defined in (2).

Proof: See Appendix I.

Numerical Examples: The integral in (4) does not have a closed form solution but it can be easily evaluated numerically using Matlab. Fig. 2 shows a plot of equivalent speed verses standard deviation for the typical values of the average vehicle speeds given in Table I. We can see that for $\sigma \neq 0$, the equivalent speed is different from the average speed. For a fixed μ, as σ increases μ_{eff} decreases, e.g. for $\mu = 90$ km/hr, $\mu_{\text{eff}} = 79.833$ km/hr for $\sigma = 27$ km/hr (a decrease of approximately 11% compared to the case of $\sigma = 0$). This important result is not reported in [9], where the average vehicle speed is taken as the equivalent speed. Substituting (4) in (3), we can determine the average number of vehicles on the highway. The results are shown in Fig. 3, which shows a plot of vehicle density $\frac{N}{L}$ in vehicles/km versus standard deviation σ (km/hr) for typical average speed $\mu = 90$ km/hr and $\lambda = 0.1$. We can see that the vehicle density is 4 vehicles/km if $\sigma = 0$. However the vehicle density increases to 4.5 vehicles/km if $\sigma = 27$ km/hr. A similar trend is observed for other values of λ and μ.

Remark 1: From (3), we can see that (a) for a fixed average speed μ, increasing σ decreases μ_{eff} and increases the number of nodes, which improves the connectivity and (b) for any σ if μ increases, then μ_{eff} increases and the number of nodes decreases, which degrades the connectivity. In particular, if the product $L\lambda$ is large then the improvement or degradation in connectivity can be significant.

Corollary 1: Consider a highway segment of length L_s carrying multiple streams of vehicles with arrival rates $\tilde{\lambda}_{kj}$ given by (1). The average number of vehicles $\tilde{N}_s$ on the highway segment, under steady state, is given by

$$\tilde{N}_s = \frac{L_s\left(\sum\limits_{k=1}^{s}\sum\limits_{j=s+1}^{S}\tilde{\lambda}_{kj}\right)}{\mu_{\text{eff}}} \tag{5}$$

where $s = 1, 2, \ldots, S-1$ is the highway segment index and μ_{eff} is given by (4).

Proof: Corollary 1 directly follows from Theorem 1 by taking into account the highway model in Section II-A. It can easily be verified by substituting the values of the highway segment indices. Note that the double summation in (5) represents the overall arrival rate for all the vehicles, belonging to any stream, present on the sth highway segment.

B. Connectivity

We study the connectivity of a highway segment using the metric of 1-connectivity. The 1-connectivity is defined as the probability that for each pair of vehicles, there exists at least 1 path connecting them. Two vehicles are considered to be connected if their distance is less than the vehicle transmitting range R. The probability that a one-dimensional stationary ad hoc network is composed of at most C clusters is given in [3]. Letting $C = 1$ and using the result from Corollary 1, we obtain the 1-connectivity of the sth highway segment as

$$P_s(\text{1-con}) = 1 - \sum_{i=1}^{m}(-1)^{i-1}\binom{\tilde{N}_s - 1}{i}(1 - i\frac{R}{L_s}) \tag{6}$$

where $m = \min\{\tilde{N}_s - 1, \lfloor\frac{L_s}{R}\rfloor\}$, R is the vehicle transmitting range, L_s is the length of the highway segment and $\tilde{N}_s$ is the average number of vehicles on the highway segment given by (5) which includes the effect of vehicle mobility. It will be shown in the next section that (6) can accurately predict the 1-connectivity of highway segments.

IV. RESULTS

In this section, we investigate the VANET connectivity and compare our numerical results with simulation results. The numerical results are obtained by numerically evaluating the analytical model presented in Section III. The simulations are carried out in Matlab by implementing the highway model presented in Section II. The total simulation time is 100,000 seconds and the simulation results are collected after a period of 500 seconds to allow the VANET to achieve steady state. For comparison, we consider the highway scenario presented in [10], i.e. we consider a $L = 10$ km highway with service points located at distances $0, 1, 3, 6$ and 10 km from the beginning of the highway (this corresponds to the case of $S = 5$ in our analytical model). The values of the various highway and traffic parameters are summarized in Table II. Note that we have 4 highway segments in the above scenario with lengths $L_1 = 1$ km, $L_2 = 2$ km, $L_3 = 3$ km and $L_4 = 4$ km, respectively.

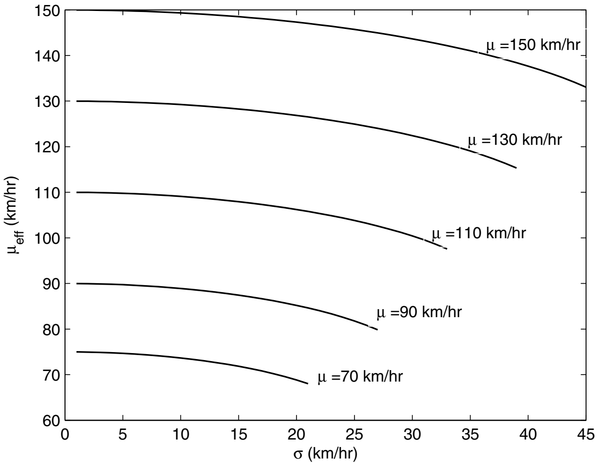

Fig. 2. Equivalent speed μ_{eff} (km/hr) verses standard deviation σ (km/hr) (calculated from (4)).

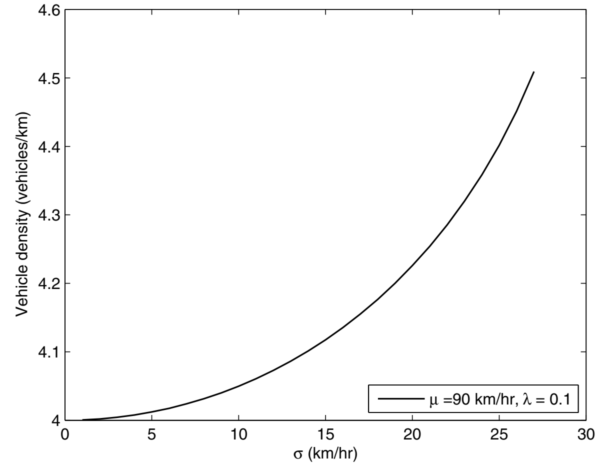

Fig. 3. Vehicle density $\frac{N}{L}$ (vehicles/km) versus standard deviation σ (km/hr) for $\lambda = 0.1$ and average speed $\mu = 90$ km/hr (calculated from (3)).

TABLE II

HIGHWAY AND TRAFFIC PARAMETER VALUES.

No.	Parameter	Value
1	Highway length	$L = 10$ km
2	No. of service points	$S = 5$
3	Service point location	$d_1 = 0$ km, $d_2 = 1$ km
		$d_3 = 3$ km, $d_4 = 6$ km, $d_5 = 10$ km
4	Arrival rates	$\lambda_1 = 0.5$, $\lambda_2 = 0.1$
		$\lambda_3 = 0.1$, $\lambda_4 = 0.2$
5	Probabilities	$\alpha_2 = \alpha_3 = \alpha_4 = 0.7$
6	Average vehicle speed	$\mu = 90$ km/hr (25 m/s)
7	Standard deviation	$\sigma = 10.8$ km/hr (3 m/s)

A. Model Validation: Average number of nodes

From (3), $\mu_{\text{eff}} = 88.704$ km/hr for $\mu = 90$ km/hr and $\sigma = 10.8$ km/hr. Substituting this value in (5), we can find the average number of nodes in each highway segment. The results are shown in Table III, rounded to the nearest integer. We can see that the numerical results are in excellent agreement with simulation results and also match the results obtained from Fig. 6 in [10]. This confirms the analytical approach which is proposed in this paper to characterise vehicle mobility.

TABLE III

AVERAGE NUMBER OF NODES IN EACH HIGHWAY SEGMENT FOR THE HIGHWAY SCENARIO OF TABLE II.

No.	Numerical results from (5)	Simulation results	Results from [10]
Segment No. 1	20	20	20
Segment No. 2	37	37	36
Segment No. 3	63	63	62
Segment No. 4	75	74	74
Whole Highway	194	194	192

B. 1-Connectivity and adaptive transmission range

Fig. 4 shows the plot of 1-connectivity of the 4 highway segments as well as the whole highway as a function of transmission range R. We can see that the simulation results are in excellent agreement with the numerical results, which confirms that (6) can accurately predict the 1-connectivity of highway segments. The results also confirm the observation in [9] and that when average number of vehicles is large, 1-connectivity of VANETs exhibits a sharp transition from zero 1-connectivity to 100% 1-connectivity for a small variation of transmission range. From Fig. 4, we can see that in order to achieve 99% 1-connectivity, the vehicles in the four highway segments require different transmission ranges, i.e. 357 m (segment 1), 467 m (segment 2), 449 m (segment 3) and 486 m (segment 4) respectively. However, a higher transmission range of 578 m is needed to achieve 99% 1-connectivity for the whole highway. These results suggest that vehicles can adaptively change their transmission range R, according to the value calculated at each service point, to ensure network connectivity while minimizing power consumption. Note that these transmission ranges are within the IEEE802.11p target transmission range for vehicular communications of 200 m $-$ 1 km [2].

C. Effect of vehicle mobility

To illustrate the effect of μ and σ on the connectivity, we consider a single highway segment of Length $L = 10$ km carrying a single stream of vehicles with arrival rate λ. Fig. 5 shows the 1-connectivity of the single highway segment as a function of transmission range R for $\lambda = 0.3$ and different speeds, using (6). We can see that for $\mu = 90$ km/hr, increasing σ from 10.8 km/hr to 27 km/hr improves connectivity by $5 - 10\%$. In addition, increasing μ leads to lower connectivity. This confirms our prediction in Remark 1. These important insights are not provided by the analysis in [9], [10].

V. CONCLUSIONS

In this paper, we have proposed a new equivalent speed parameter, to accurately characterise the effect of vehicle mobility on connectivity of VANETs. We derived an analytical expression for the average number of nodes on a highway segment, which was used to determine the VANET 1-connectivity. It has been shown that (a) equivalent speed parameter is different from the average vehicle speed and decreases as

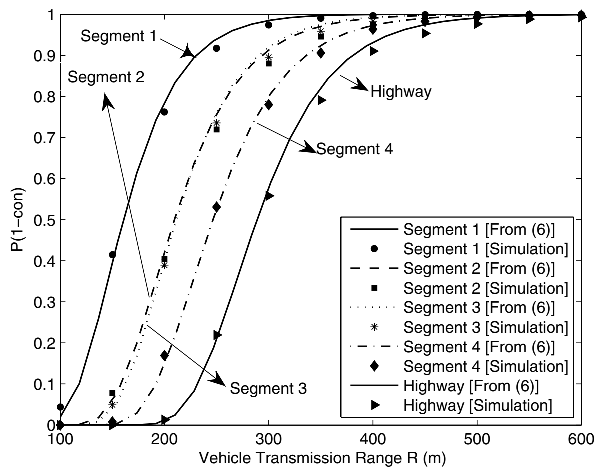

Fig. 4. 1-connectivity of the highway segments versus vehicle transmission range R for the highway scenario given in Table II.

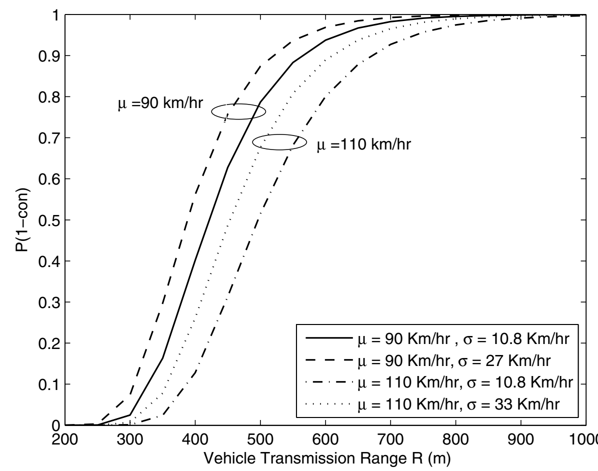

Fig. 5. 1-connectivity of a single highway segment versus vehicle transmission range R for $\lambda = 0.3$, $L = 10$ km and different speeds.

the variance of the vehicle speed increases, (b) increasing the average vehicle speed increases the equivalent speed which degrades connectivity while increasing the standard deviation of the vehicle speed decreases the equivalent speed which improves connectivity and (c) vehicles can adaptively choose their transmission range to ensure network connectivity in highway segments while minimising power consumption.

APPENDIX I
PROOF OF THEOREM 1

First consider the case of a highway segment of length L carrying a single stream of vehicles with arrival rate λ and all vehicles are moving with the same constant speed μ, i.e. the vehicles do not overtake each other. In this case, it is well known that the inter-vehicle distance z follows an exponential distribution given by $f_z(z) = \frac{\lambda}{\mu}\exp(\frac{\lambda}{\mu}z)$ [5]. Thus the average number of vehicles N on the highway segment is $N =$ length of highway $\times$ vehicle density $= L \times \frac{1}{E\{z\}} = \frac{L\lambda}{\mu}$, where $E\{\cdot\}$ denotes expectation operator.

Next consider the case of a highway segment of length L carrying a single stream of vehicles with arrival rate λ and the vehicle speeds are chosen from a truncated Gaussian PDF as given by (2). In this case, vehicles with faster speeds can overtake vehicles with slower speeds. This makes the task of finding the probability density function of the inter-vehicle distance complicated and nontrivial.

To overcome this problem, we propose to view a single stream of vehicles with Gaussian distributed speeds as a *sum of infinite number of parallel virtual streams*, where each virtual stream is comprised of vehicles with a particular constant speed $V_{\min} \le v \le V_{\max}$. The arrival rate of vehicles in each virtual stream is determined by the probability of occurrence of that particular speed v as governed by the truncated Gaussian PDF. Using this novel approach, the average number of vehicles N can be written as

$$N = L \times \int_{V_{\min}}^{V_{\max}} \frac{\lambda \hat{f}_V(v)}{v} dv \qquad (7)$$

where $\hat{f}_V(v)$ is given by (2). Substituting values from (2) into (7) and rearranging leads to (3) and (4).

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