Modified Constant Modulus Algorithm for joint blind equalization and synchronization

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***Abstract*—The development of low-complexity blind techniques for equalization, timing and carrier offset recovery is of enormous importance in the design of high data rate wireless systems. In this paper, we propose a practical solution for blind equalization, timing recovery and small carrier offset correction in slowly-fading frequency selective wireless communication channels. We extend the Modi ed Constant Modulus Algorithm (MCMA) to handle the timing offset parameter. We propose a single objective function to achieve equalization, timing and carrier recovery without the need of any training sequences. Our algorithm achieves 5-10 times faster convergence, compared to previous research, for the Mean Square Error (MSE) at the equalizer output due to the nely tuned step sizes and updating the equalizer taps and timing offset jointly to minimize the mean dispersion and inter symbol interference. Our results show that the proposed technique can successfully handle synchronization and combat the frequency selectivity of the wireless channel.**

I. Introduction

Adaptive equalization techniques are a topic of immense theoretical and practical value with growing applications in areas of digital and wireless communications [1]. The use of initial training sequences in adaptive equalization gives rise to significant overhead with data rate reduction and sometimes may become unrealistic or impractical. For instance, no training signal may be available to receivers in military communication scenarios and defence applications. In a multicast or broadcast system, it is highly undesirable for the transmitter to engage in a training session for a single user by temporarily suspending its normal transmission to a number of other users. Consequently, there is a strong and practical need for blind equalization without training [1].

There are two main approaches to blind equalization. The first category is stochastic gradient algorithms which process one data sample or a small block of data at a time and iteratively minimize a chosen cost function [2]. The second category is statistical methods which use sufficient stationary statistics collected over a large block of received data [2]. A limitation of statistical methods is that they require the channel to be constant over the block of received data. This requirement is generally not fulfilled by real-world wireless channels which cannot be guaranteed to be static over long time intervals. Hence stochastic gradient algorithms can be used to adapt in slowly fading wireless channels while working on a symbol by symbol basis. The issue of slow convergence in stochastic gradient algorithms is well known and has been addressed by many authors recently to make it faster and practical in various scenarios [3]–[7].

Due to its simplicity, Constant Modulus Algorithm (CMA) is the most commonly used stochastic gradient algorithm from practical implementation point of view [8]. In CMA, a separate loop is required for tracking the channel and the carrier phase offset respectively, which can be avoided using Modified CMA (MCMA) cost function [9]. The convergence speed of MCMA is, however, slower than CMA. This can be improved in a number of ways, e.g. by using a combination of MCMA and Super Exponential algorithm [4], by joining MCMA and Decision Directed (DD) tracking with variable step size adjustment [3], [6], by using decision feedback equalizer [7] and also by appending DD-Least Mean Square algorithm with CMA [5]. All of these algorithms assume perfect timing recovery at the receiver, which is never possible in realistic communication systems.

To the best of our knowledge, the important problem of joint equalization, carrier and timing recovery for wireless communications has only been targeted by a few authors [10], [11]. In [10], an algorithm is presented for joint equalization, carrier and timing recovery which is suited for cable modem transmission and cannot be directly applied to wireless communication scenarios. In [11], the task is accomplished by using digital phase-locked loop (PLL). However the acquisition speed of a PLL is very slow, which makes it impractical for time varying wireless channels. Most of the previous work in [2]–[9] focuses on the subset of these problems, either joint equalization and carrier recovery or blind equalization alone. Hence there is a need for the development of practical, low-complexity, blind techniques for equalization, timing recovery and carrier offset correction.

In this paper, we propose a technique for blind timing recovery, equalization and carrier offset recovery jointly without any aid of training, which is applicable for slowly-fading, frequency selective wireless channels. We modify MCMA algorithm of Oh et al. [9] to handle timing offset parameter. We propose a modified single objective function to achieve equalization, carrier and timing recovery. Our simulation results show very fast convergence compared to the results of all the above mentioned papers, i.e., [3]–[5]. The major contributions of this paper, in comparison to previous research, are as follows:-

- We propose an algorithm, based on joint minimisation of a single objective function, to accomplish equalization, timing recovery and small carrier offset recovery without the need of any training sequences and phase locked loop.

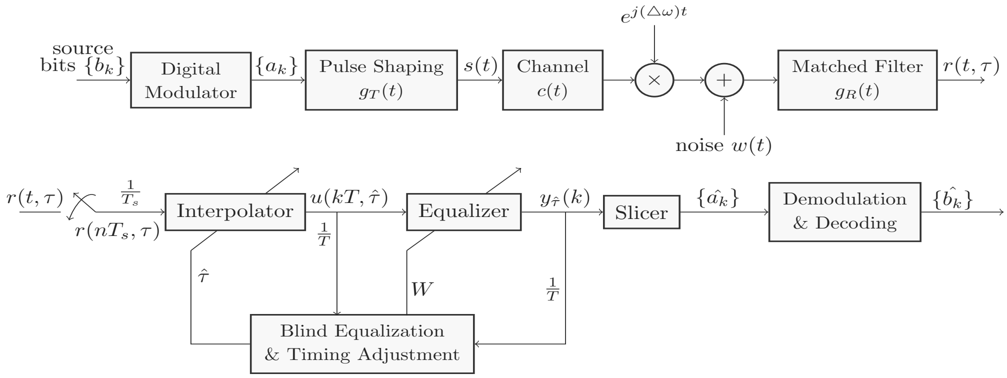

Fig. 1. Baseband Communication System for Blind Synchronization and Equalization.

- We show that the proposed algorithm has $5-10$ times faster convergence of Mean Square Error compared to the previous research, while providing additional capability of handling synchronization. This is due to the finely tuned step sizes and updating the equalizer taps and timing offset jointly to minimize the mean dispersion and inter symbol interference.

Finally, we outline the algorithm in a step-wise manner for an ease of practical implementation.

II. System Model

The baseband model of the proposed system for a single source and a single destination is shown in Fig. 1. At the transmitter, the sequence of information bits b_k is applied to constellation mapper, which converts b_k into a complex valued sequence a_k of data symbols, taken from two dimensional constellation and the two components (in-phase and quadrature) are transmitted by amplitude modulating two quadrature carrier waves. Such a modulation scheme can be treated in a concise manner by combining in-phase and quadrature components into complex valued symbols. The data symbols enter the transmit filter with the impulse response $g_T(t)$ and the resulting transmit signal $s(t)$ is given by

$$s(t) = \sum_{k=-\infty}^{\infty} a_k g_T(t-kT) \quad (1)$$

where $1/T$ is the symbol rate, i.e., the rate at which the data symbols are applied to the transmit filter, g_T is the baseband pulse at the transmitter which is matched to the receiver filter g_R, i.e., $g_T(t) = g_R(-t)$. The signal at the output of the receive filter is given by

$$r(t,\tau) = \sum_{k=-\infty}^{\infty} a_k h(t-\tau T-kT)e^{j\phi(t)} + v(t), \quad (2)$$

where $h(t) = g_T(t) * c(t) * g_R(t)$ represents the overall baseband impulse response, $c(t)$ is the baseband multipath channel impulse response which introduces attenuation and phase distortion in the signal, $\phi(t) = (\triangle\omega)t$ represents the frequency offset between the transmitter and receiver oscillators, which can be visualized as the time varying phase offset and results in continuous spinning of constellation with time and '$*$' represents the convolution operator. The constant phase offset between the transmitter and receiver carriers can be merged with the channel induced phase offset ϕ_o which rotates the transmitted constellation, τ, normalized by the symbol duration T, is the fractional unknown timing offset ($|\tau| \leq \frac{1}{2}$) between the transmitter and receiver filters and $v(t)$ is the complex filtered noise $v(t) = g_R(t) * w(t)$ with variance σ_v^2, where $w(t)$ is the zero mean stationary, white, and complex Gaussian process.

After pulse shaping, the signal is sampled with some timing offset since the receiver does not know the exact sampling point corresponding to maximum Signal to Noise Ratio (SNR). The receive filter output is oversampled by the factor Q such that the oversampling period $T_s = T/Q$, so (2) becomes

$$r(nT_s,\tau) = \sum_{k=-\infty}^{\infty} a_k h(nT_s-\tau T-kT)e^{j\phi(nT_s)} + v(nT_s), \quad (3)$$

where n is the sampling index, $\phi(nT_s) = 2\pi(\triangle f)nT_s = 2\pi(\triangle f/F_s)n$, $\triangle f$ is the frequency offset in Hz and $\triangle f/F_s$ is the digital frequency offset in cycles/sample [12]. The noise samples $v(nT_s)$ are assumed to be statistically independent of the input symbols a_k. Sampling the received signal at wrong instant (not at the maximum eye opening) or any jitter in the sampler introduces Inter Symbol Interference (ISI) and results in the reduction of noise margin. Zero ISI can be achieved only by sampling at exact instant kT, for k_{th} symbol applied to the transmit filter.

The problem of joint synchronization, equalization and small carrier offset recovery using blind algorithms is to:

1) Estimate the timing offset τ.
2) Estimate the equalizer taps to equalize the multipath channel $c(t)$.
3) Mitigate of constant channel induced phase offset ϕ_o.
4) Mitigate small time varying phase offset $\phi(k)$.

The proposed solution is presented in the next section.

III. Proposed Receiver

In this section, we present the receiver design, which is dedicated to compensate for all the distortions (timing offset, frequency and phase offset and multipath channel distortion) jointly without any aid or known preamble.

A. Interpolator

Simple linear interpolation is used, which works on estimated timing offset $\hat{\tau}$ provided by the Blind Equalization and Timing Adjustment block of Fig. 1. Instead of linear interpolators, Lagrange or higher degree polynomial interpolators can also be used [13]. However the resulting performance improvement over linear interpolators is very minute and comes at a cost of greater computational complexity. Hence we use linear interpolators as a good compromise between performance and complexity. Linear interpolator has a delay of $Q-1$ samples which can be stored in its delay line before preforming interpolation. Depending on $\hat{\tau}$ and Q, the linear interpolation [13] between two appropriate samples is given by

$$u(nT_s,\hat{\tau}) = r(nT_s,\tau) + \mu[r((n+1)T_s,\tau) - r(nT_s,\tau)], \quad (4)$$

where $\mu = \hat{\tau} \times Q$. The sampling rate after interpolation is reduced to $1/T$ for baud spaced equalization, i.e., interpolation is performed on interleaved samples.

B. Blind Equalization and Timing Adjustment

For multipath channels $c(t)$, each received signal sample $r(nT_s,\tau)$ depends on multiple transmitted symbols a_k and the resulting ISI is mitigated through multitap equalizer W. Let us denote the equalizer output $y(kT,\hat{\tau})$ as $y_{\hat{\tau}}(k)$ as shown in Fig. 1. The carrier offset recovery and estimate of the parameters $\hat{\tau}$ (the timing offset) and W (the equalizer tap weight vector) are obtained by minimizing the mean dispersion of the sampler output and using the modification in CMA cost function [14]. Introducing new parameter, $\hat{\tau}$, to cost function of Oh and Chin [9], the modified objective function can be written as

$$J_k(\hat{\tau},W) = J_{R,k}(\hat{\tau},W) + J_{I,k}(\hat{\tau},W) \quad (5)$$

where $J_{R,k}(\hat{\tau},W)$ and $J_{I,k}(\hat{\tau},W)$ are the cost functions for the real part $\Re\{y_{\hat{\tau}}(k)\} = y_{R,\hat{\tau}}(k)$ and imaginary part $\Im\{y_{\hat{\tau}}(k)\} = y_{I,\hat{\tau}}(k)$ of the equalizer output respectively, and are defined as

$$J_{R,k}(\hat{\tau},W) = E\{(|y_{R,\hat{\tau}}(k)|^2 - \gamma_R)^2\} \quad (6)$$

$$J_{I,k}(\hat{\tau},W) = E\{(|y_{I,\hat{\tau}}(k)|^2 - \gamma_I)^2\}, \quad (7)$$

where $|\cdot|$ is the modulus operator, $E\{\cdot\}$ represents the expected or mean value and γ_R and γ_I are constants for the real $\Re\{a_k\} = a_{R,k}$ and imaginary $\Im\{a_k\} = a_{I,k}$ parts of the input data sequence respectively. These constant factors are proportional to the kurtosis of the input sequence and are determined as [8]

$$\gamma_R = \frac{E\{|a_{R,k}|^4\}}{E\{|a_{R,k}|^2\}} \quad (8)$$

$$\gamma_I = \frac{E\{|a_{I,k}|^4\}}{E\{|a_{I,k}|^2\}} \quad (9)$$

where input data sequence a_k is assumed to be stationary and non gaussian process and $a_k = a_{R,k} + ja_{I,k}$, with i.i.d. real and imaginary parts [2].

(8) and (9) are determined subject to the constraint that the average gradient of the cost function (5) with respect to the tap weight vector W is zero when the channel is perfectly equalized. The objective cost function given by (5) is obtained by using $p = 2$ (dispersion order) in the generalized Constant Modulus Godard's cost function $E[(|y_{\hat{\tau}}(k)|^p - \gamma_p)^2]$ for $\gamma_p = E\{|a_k|^{2p}\}/E\{|a_k|^p\}$ [8]. The value of $p = 2$ is chosen because the practical digital implementation with finite length arithmetic (fixed point implementation) suffers from precision or overflow problems for large p [8]. Godard also demonstrated that the cost function can be applied to non-constant modulus signals such as rectangular QAM constellation [8].

Let $W(k) = \{w_0, w_1, \ldots, w_{N-1}\}^t$ be the adjustable tap weights for N tap equalizer where superscript $(\cdot)^t$ denotes the transpose of a vector. If $X(k) = \{u_{\hat{\tau}}(k), u_{\hat{\tau}}(k-1), \ldots, u_{\hat{\tau}}(k-N+1)\}^t$ are interpolated samples in the delay line of equalizer then equalizer output is

$$y_{\hat{\tau}}(k) = X^t(k)W(k) \quad (10)$$

The estimated equalizer weight vector $W(k)$ is fed to the equalizer by Blind Equalization and Timing Adjustment block which uses the steepest descent type method of stochastic gradient algorithm and optimizes the cost function (5) to adapt the tap weight vector and timing offset [2]. In this case, the modified steepest descent update equations are

$$W(k+1) = W(k) - \mu_w \bigtriangledown_w J_k(\hat{\tau},W) \quad (11)$$

$$\hat{\tau}(k+1) = \hat{\tau}(k) - \mu_{\hat{\tau}} \bigtriangledown_{\hat{\tau}} J_k(\hat{\tau},W), \quad (12)$$

where μ_w and $\mu_{\hat{\tau}}$ are small positive step sizes which control the convergence of parameter update equations and $\bigtriangledown_w J_k(\hat{\tau},W)$ and $\bigtriangledown_{\hat{\tau}} J_k(\hat{\tau},W)$ are the gradients of the cost function (5) with respect to equalizer tap weights and timing offset respectively. Solving these gradients and then substituting in (11) and (12) results in

$$W(k+1) = W(k) - \mu_w e_w(k) X^H(k), \quad (13)$$

$$\hat{\tau}(k+1) = \hat{\tau}(k) - \mu_{\hat{\tau}}(|y_{\hat{\tau}}(k)|^2 - \gamma)\frac{\partial |y_{\hat{\tau}}(k)|^2}{\partial \hat{\tau}}, \quad (14)$$

where the superscript $(\cdot)^H$ represents the conjugate transpose (Hermitian) operator and the error signal for weight update $e_w(k) = e_{w,R}(k) + je_{w,I}(k)$ is given as

$$e_{w,R}(k) = (|y_{R,\hat{\tau}}(k)|^2 - \gamma_R)\, y_{R,\hat{\tau}}(k) \quad (15)$$

$$e_{w,I}(k) = (|y_{I,\hat{\tau}}(k)|^2 - \gamma_I)\, y_{I,\hat{\tau}}(k) \quad (16)$$

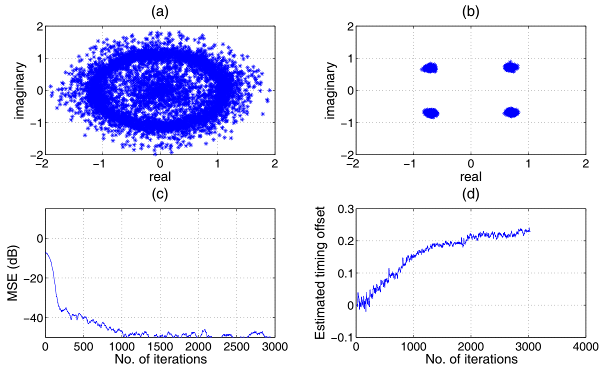

Fig. 2. Results for QPSK with multipath channel, fractional timing phase offset $(\tau) = -0.4$, frequency offset $(\triangle f/F_s) = 10^{-4}$ and SNR = 25 dB. (a) Channel output constellation. (b) Equalized output constellation. (c) MSE between equalizer output and transmitted symbols. (d) Timing offset recovery.

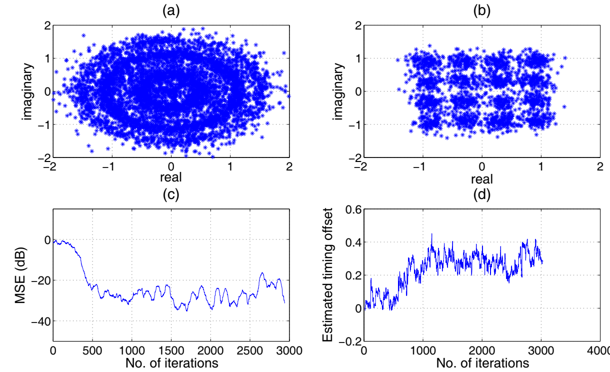

Fig. 3. Results for 16-QAM with multipath channel, fractional timing phase offset $(\tau) = -0.4$, frequency offset $(\triangle f/F_s) = 10^{-4}$ and SNR = 25 dB. (a) Channel output constellation. (b) Equalized output constellation. (c) MSE between equalizer output and transmitted symbols. (d) Timing offset recovery.

The derivative of $|y_{\hat{\tau}}(k)|^2$ in (12) is computed using Euler's approximation by evaluating $y_{\hat{\tau}}(k)$ at $\hat{\tau} + \delta$ where δ is very small increment and then using the following equation

$$\frac{\partial |y_{\hat{\tau}}(k)|^2}{\partial \hat{\tau}} \simeq \frac{|y_{\hat{\tau}+\delta}(k)|^2 - |y_{\hat{\tau}}(k)|^2}{\delta} \qquad (17)$$

where δ is 0.0001 in our case.

Intuitively, the algorithm tries to move the the real part of the equalizer output to reside on the points of value $+\sqrt{\gamma_R}$ or $-\sqrt{\gamma_R}$ on the real axis instead of a circle. Similarly, imaginary part of the equalizer output is moved to lie on the points of value $+\sqrt{\gamma_I}$ or $-\sqrt{\gamma_I}$ on imaginary axis. Since, the cost function employs both modulus and phase of the equalizer output, channel induced phase offset ϕ_o and small carrier phase offset $\phi(k)$ are corrected along with blind equalization without the need of separate decision directed phase tracking loop [9]. Finally, the slicer makes a hard decision on equalizer output generating $\hat{a_k}$ which are demodulated and decoded to produce the output bits $\hat{b_k}$.

C. Algorithm

The whole blind setup can be summarized in the form of the following algorithm.

Proposed Algorithm

<u>Initialization</u>:

Initialize equalizer tap weight vector W with zeros and substitute 1 for the central tap. Initialize estimated timing offset $\hat{\tau} = 0$ and vector X and X_d of equalizer length with zeros. Let u be the resulting interpolated sample.

<u>Loop Processing</u>:

1) Use (4) for linear interpolation on the received symbols.
   IF $0 \le \hat{\tau} \le 0.5$
   u = interpolation between current and the next sample.
   ELSE IF $-0.5 \le \hat{\tau} \le 0$
   u = interpolation between current and the previous sample
   END
2) Update X by right shifting by one sample and replacing the first sample by u.
3) Repeat steps 1 & 2 for X_d using $\hat{\tau} + \delta$ in place of $\hat{\tau}$.
4) Calculate $y_{\hat{\tau}} = X^t W$ and $y_{\hat{\tau}+\delta} = X_d^t W$.
5) Evaluate (17).
6) Update W and $\hat{\tau}$ using (13) and (14) respectively.

The processing is done on every received symbol to handle small carrier offset and track the revolving constellation. The complexity of the algorithm can be reduced in case of static multipath channels with no carrier offset by stopping the parameter updates, after the convergence has been achieved.

IV. Simulation Results

The simulations are carried out in MATLAB. We have shown the results for static frequency selective wireless channel but they can be extended easily for slow fading channel due to the symbol by symbol processing and adaptive nature of algorithm. We consider QPSK and 16-QAM modulations, which are widely-considered constellations. Simulation results are evaluated at 25 dB SNR under two different multipath channels to validate the robustness of the algorithm. The oversampling factor Q is set to 2 for implementing linear interpolation. Root raised cosine filters are used for transmitter pulse shaping and receiver matched filtering with roll off factor set to 0.25. The carrier frequency offset $(\triangle f/F_s)$ is set to 10^{-4} [9]. We have modeled the timing offset τ produced by the sampler by filtering the channel output with offset exhibiting matched filter samples. Mean Square Error (MSE) or average instantaneous squared error between the equalizer output and the reference transmitted symbol over 60 realizations is calculated as the performance metric. The

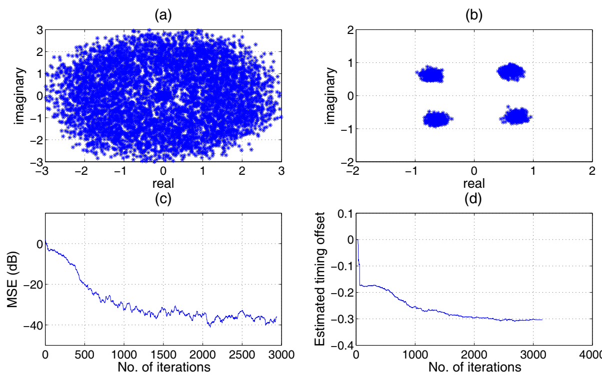

Fig. 4. Results for QPSK with multipath channel, fractional timing phase offset $(\tau) = 0.4$, frequency offset $(\triangle f/F_s) = 10^{-4}$ and SNR = 25 dB. (a) Channel output constellation. (b) Equalized output constellation. (c) MSE between equalizer output and transmitted symbols. (d) Timing offset recovery.

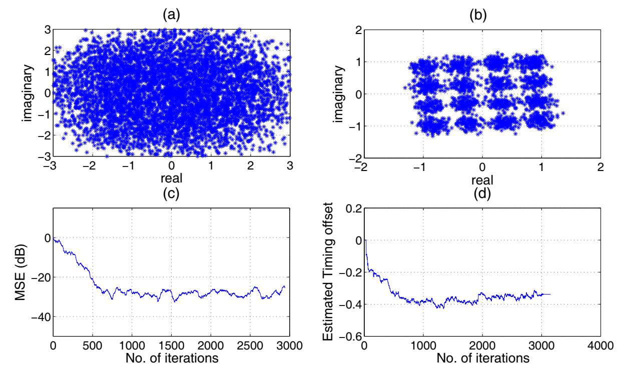

Fig. 5. Results for 16-QAM with multipath channel, fractional timing phase offset $(\tau) = 0.4$, frequency offset $(\triangle f/F_s) = 10^{-4}$ and SNR = 25 dB. (a) Channel output constellation. (b) Equalized output constellation. (c) MSE between equalizer output and transmitted symbols. (d) Timing offset recovery.

values of the step sizes are selected by hit and trial method and are shown in Table I

TABLE I
UPDATE STEP SIZES

Modulation	Chan-1		Chan-2	
	$\mu_{\hat{\tau}}$	μ_w	$\mu_{\hat{\tau}}$	μ_w
QPSK	$1e-2$	$5e-2$	$1e-2$	$5e-2$
16-QAM	$5e-3$	$5e-2$	$5e-3$	$5e-2$

A. Results for Channel 1

The first channel (Chan-1) is taken from [4] and [15], which define the multipath channel impulse response as

$$C(z) = (0.4-0.6z^{-1}+1.1z^{-2}-0.5z^{-3}+0.1z^{-4})e^{j\pi/4}/1.41 \tag{18}$$

The results for Chan-1 are shown in Fig. 2 and 3 for QPSK an 16-QAM, respectively. The timing phase offset τ is set to -0.4 to introduce ISI. The equalizer is selected as a 7 tap filter with central tap initialized to 1. The values for step sizes $\mu_{\hat{\tau}}$ and μ_w are shown in Table I.

Fig. 2(a) and Fig. 3(a) shows the channel output. We can see that amplitude distortion and ISI due to multipath channel and timing offset has deteriorated the transmitted symbols. The circular appearance of the constellation is due to carrier frequency offset. Fig. 2(b) and Fig. 3(b) shows the equalized output. MSE performance and estimated timing offset $\hat{\tau}$ are shown in Fig. 2(c), 2(d) and Fig. 3(c), 3(d) respectively. $\hat{\tau}$ converges to $+0.25$ and $+0.3$ approx. for QPSK and 16-QAM respectively to mitigate the introduced negative offset. Since there is also an ISI due to the multipath channel, the estimated offset converges to a different value instead of 0.4 to mitigate the resultant channel dispersion. The MSE converges to the desirable level of -50 dB for QPSK and -30 dB for 16-QAM, in just 500 iterations which is a great improvement on the results shown by Zhang et al. [4], where the convergence is achieved after 5000 iterations under same channel with no joint timing recovery. This is accomplished due to the finely tuned step sizes and updating equalizer taps and timing offset jointly to minimize the mean dispersion and ISI.

B. Results for Channel 2

The second channel (Chan-2) is taken from [3], which is a multipath channel, defined in Signal Processing information database SPIB(#2) [16]. The results are shown in Fig. 4 and. 5 for QPSK an 16-QAM modulations respectively. The timing phase offset τ is set to $+0.4$ to introduce ISI. The equalizer is selected as a 16 tap filter with central tap initialized to 1. The values for step sizes $\mu_{\hat{\tau}}$ and μ_w are shown in Table I. Sub-figures (a)-(d) demonstrate the same graphs as described for Chan-1. $\hat{\tau}$ converges to -0.3 and -0.35 approximately. for QPSK and 16-QAM respectively to mitigate the resultant channel dispersion. The MSE converges to an acceptable level of -35 dB for QPSK and -30 dB for 16-QAM, in just 650 iterations which is a considerable improvement on the results shown by Ashmawy et al. [3], where the convergence is achieved after 3000 iterations under same channel with no joint timing recovery. This is accomplished due to the finely tuned step sizes and updating equalizer taps and timing offset jointly to minimize the mean dispersion and ISI.

V. CONCLUSION

We have accomplished blind timing recovery, equalization and small carrier offset recovery jointly without any training sequence. We have modified CMA and extended the objective function to handle another parameter of timing phase offset. We have validated the robustness of algorithm by showing simulation results under two different multipath channels. We have shown the fast convergence for MSE between the equalizer output and the expected value. Our simulation results show approximately 5 to 10 times improvement in MSE performance compared to the previous research papers, while using their channel and handling synchronization in addition. Moreover, we have summarized the algorithm for an ease of implementation. In future, the results can be extended for wireless block fading channel and handling larger carrier frequency offsets by data reuse.

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