Problem Dealt with : K-SAT. (k literals per clause) ($k=3$ in experiment), M clauses. N boolean variables ($N=10^7$)

$\alpha = \frac{M}{N}$, α_c separates region almost all formulas are SAT / all are UNSATfiable

for $k=3$, $\alpha_c \approx 4.267$ (this paper up to 4.24). Difficulty due to $\exists$ clustering phenomenon

when $\alpha \to \alpha_c$ $\boxed{\text{Two step iterates between}}$ (1) survey/message propagation which are surveys over clusters of the ordinary messages. (2) use the probabilistic info to fix var.

Experiment : 3-SAT. $N \sim 10^7$

choose formula. choose k-tuples of var at random (with no repetition)

negate var with $P_r = 0.5$. ε in SP $= 10^{-3}$ 用小无 significant improve

① $\alpha < \alpha_d \simeq 3.9$ converge to a set of trivial msg $\delta\eta_{a\to i} = 0$ for all a-i edges. all var are under-constrained. (no constrained cluster, i.e. some var in the cluster is fixed)

② $\alpha \in (3.9, 4.3)$. converges to unique fixed-point set of non-trivial msg, independent from initial conditions, & large fraction of msg are in (0,1). $\exists$ constrained clusters

outperform existing algo.

③ For small N ($N=1000$) often SP doesn't converge. But more converge Prob. with $N\uparrow$ (fix α, <4.3)

Results. A single run of decimation; without restart.

fail = not converge or simplified sub-formula found by SID isn't solved by ~~SA~~ WalkSAT

so NOT necessarily UNSAT

→ Performance ↑ with $N\uparrow$. $f\downarrow$

→ larger α, more failure.

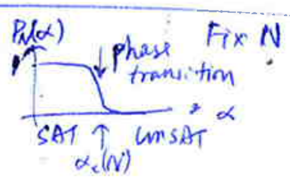
→ convergence time of SP basically doesn't grow with N. (like $\log N$)

If fix one var each step. then $O(N \cdot N \cdot \log N)$

N vars to fix, SP, SP convergence steps

Still some small gap (4.24. 4.267)

$P_N(\alpha)$: randomly generated formula is SAT : $\downarrow$ with α



define $\alpha_c(N)$ = root of $P_N(\alpha_c(N)) = \frac{1}{2}$.

$\lim_{N\to\infty} \alpha_c(N) = 4.27$ empirically near $\alpha_c(N)$ is difficult to solve

$\lim_{N\to\infty} P_N(\alpha) = 1$ for $\alpha < \alpha_{lb}$ For 3-SAT. $= 3.42$

$\lim_{N\to\infty} P_N(\alpha) = 0$ for $\alpha > \alpha_{ub}$ $= 4.506$

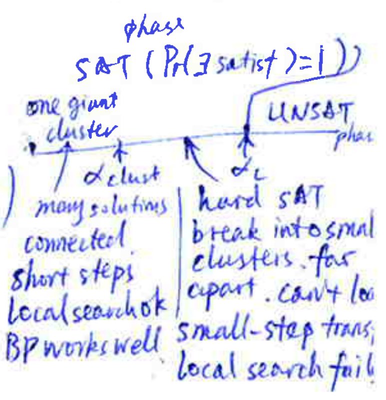
Following appears when $N \to \infty$ with α fixed :

1. $\exists$ phase transition critical value α_c. ($= 4.267$ for 3-SAT)

2. α_{clust} [illegible]

cavity method, no rigorous, except K-XOR-SAT

no rigorous proof of SP convergence

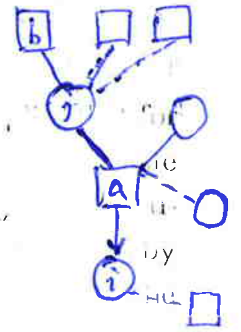


① Warning Propogation: only from ~~factor~~ function nodes to nodes $\square \to \bigcirc$ Find SAT Assign or UNSAT.

A warning $u_{a\to i}=1$ can be interpreted as a message from function node a, telling variable i that it should adopt the correct value to satisfy clause a. $\in\{0,1\}$.

So send 1 iff all neighbors of a (except i) don't want to satisfy a.

$$\text{iff } \forall j\in V(a)\backslash i,\ J_j^a \neq \mathrm{sgn}\Big(\underbrace{\sum_{b\in V(j)\backslash a} J_j^b u_{b\to j}}_{\text{average of what } j\text{'s neighbors want } j \text{ to be}}\Big)$$



Ref ~~看~~ WP Algo Page 5 (random initialization, tmax converge/not) ~~&~~ & WID Algo Pg 6 (~~见第3页~~ see page 3)

conflicting msg

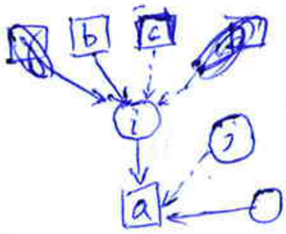
For i: If $\sum_{b\in V_+(i)} u^*_{b\to i}>0$ & $\sum_{b\in V_-(i)} u^*_{b\to i}>0$ then fail (UNSAT) return UNCONVERG SAT UNSAT (assignment)

Otherwise if ~~一正一零~~ (one positive one zero) then fix to the 正. ~~若全零~~ (If all zero). choose one unfixed arbitrarily assign

Thm 1. If factor graph is a tree, then WP converges to a unique set of fixed point warning msg. independent of initial warnings. If at least one of the contradiction numbers $c_i=1$, then the problem itself is UNSAT. Otherwise, SAT.

② Belief Propogation. $\square \leftrightarrow \bigcirc$

Gives, for tree factor graphs, the total # of SAT assignments. the fraction of SAT assignments when a variable x_i = true



1° $\underset{P(x_i\mid a \text{ is absent})}{\mu_{i\to a}(x_i)} \propto \prod_{b\in V(i)\backslash a} \underset{P(b \text{ is satisfied}\mid x_i)}{\mu_{b\to i}(x_i)}$

$$P(x_i\mid a\ \text{absent}) = P(x_i\mid bc\bar{a}) = \frac{P(bc\mid x_i\bar{a})P(x_i\bar{a})}{P(bc\bar{a})} = \frac{P(b\mid x_i)P(c\mid x_i)P(x_i)}{P(bc)}$$

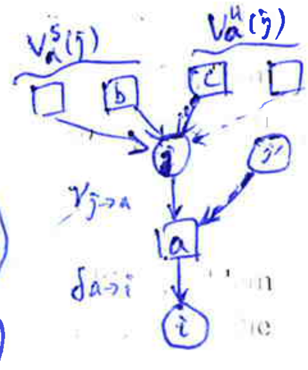
2° $\mu_{a\to i}(x_i) = \sum_{x_j:j\neq i} f_a(X)\prod_{j\in V(a)\backslash i}\mu_{j\to a}(x_j)$

$\mu_{a\to i}(x_i)$: P(a's satisfied | x_i)

$$\sum_{x_j:j\neq i} f_a(X)\,P(X\mid x_i) = \sum_{x_j:j\neq i} f_a(X)\prod_{j\in V(a)\backslash i}\mu_{j\to a}(x_j)$$

$\mu_{j\to a}(x_j)$: $P(x_j\mid a\text{'s absent})$

well if $\alpha<\alpha_{clust}$ → independence factorization



Reparameterize: • $\gamma_{j\to a}\in[0,1] = P(x_j \text{ violates } a\mid a \text{ absent})$

• $\delta_{a\to i} := \prod_{j\in V(a)\backslash i}\gamma_{j\to a}$ ← P(all var in a violate a) except i, i.e. a should send a warning to i

$$\gamma_{j\to a} = \frac{P\binom{\text{no warning}}{\text{from } V_a^s(j)}}{P\binom{\text{no warning}}{\text{from } V_a^s(j)} + P\binom{\text{no warning}}{\text{from } V_a^u(j)}}$$

$$P\binom{\text{no warning}}{\text{from } V_a^s(j)} = \prod_{b\in V_a^s(j)}(1-\delta_{b\to j})$$

$\delta_{b\to j}$: b sends warning to j

$\delta_{a\to i} = \prod_{j\in V(a)\backslash i}\gamma_{j\to a}$ (set to 1 if a is leaf with single neighbor i)

If $\alpha>\alpha_{clust}$ then BP factor doesn't hold globally but holds if one restricts prob. space to given cluster α. So BP message will converge to statistical meanings under cluster α. ~~接第3页(第一行)~~

③ survey propogation: also heuristic to loopy graphs, no guarantee of convergence
more efficient than WP. BP.

In a cluster α: some var constrained to 0, some to 1, some unconstrained $*$
so discribe a cluster by N-dim vect $X^\alpha \in \{0,1,*\}^N$. It discards alot of info. e.g.
$x_i^\alpha = *$ lost the info on the fraction of assignments in the cluster α where $x_i = 0$
接 Eq 34-37. Page 15. change i to j

warning from $b \in V_a^s(j)$	warning from $V_a^u(j)$			normalize
nil	nil	$\pi^0_{j\to a}$		W_j^+
nil	≥ 1	$\pi^u_{j\to a}$	violates a	W_j^-
≥ 1	nil	$\pi^s_{j\to a}$		$W_j^{(0)}$
≥ 1	≥ 1	contradictory		

So $\gamma_{j\to a} = \pi^u_{j\to a} / (\pi^0_{j\to a} + \pi^u_{j\to a} + \pi^s_{j\to a})$. $\delta_{a\to i} = \prod_{j\in V(a)\setminus i} \gamma_{j\to a}$

$P(x_j$ violates a | a absent)

SID: check $|W_j^+ - W_j^-|$ pick $j^* = \arg\max_j |W_j^+ - W_j^-|$. $x_{j^*} \leftarrow 1$ if $W_{j^*}^{(+)} > W_{j^*}^{(-)}$

① · all are trivial (all $\delta = 0$) → output simplified sat-formula. → walkSAT → found SAT / not find SAT → ① no contradiction; contradiction → probably UNSAT

· some $\delta \neq 0$ → 求 $W_i^{(+)(-)(0)}$, $j^* = \arg\max_j |W_j^+ - W_j^-|$. fix x_{j^*} or fix a proportion of the top few bias variables. → ①

① SP → converge; unconverge → Ret UNCONV.

WID.
① → WP → not converge → Ret NOT_CONV.
收敛 → ≥ 1 contradiction → UNSAT.
→ ≥ 1 One $H_i \neq 0$ → Fix all $H_i \neq 0$ ($H_i \gtrless 0 \Rightarrow x_i = {1 \atop 0}$) clean → ①
→ all $H_i = 0$ → Fix an unfixed var to arbitrary. clean → ①

message passing proctocal
update simultaneously all δ belonging to the same clause,
order of clause is a random permutation chosen at each iteration step

Distance between two assignments $x_i, y_i = \sum_i (x_i - y_i)^2$, $<\epsilon$, then connected. path connected nodes form a clus

SP msg ~~含义~~ meaning:
If in every SAT assignment of cluster α, all var $x_j \in V(a)\setminus i$ don't satisfy a, then a warning $u^\alpha_{a\to i} = 1$ is passed $a \to i$. The SP message along this edge is the survey of these warnings.
when one picks up a cluster α at random $\delta_{a\to i} = \sum_\alpha u^\alpha_{a\to i} / |\#\text{cluster}|$. So SP msg gives prob. that there's a warning from $a \to i$ in a random cluster. ③ also $W_i^{(+)(-)(0)}$ has meaning
② complexity log # constrained clusters of SAT assignments / fraction of constrained clusters when x_i is frozen pos/neg / unconstrained Pg 17
underconstrained, biased