Plücker Basis Vectors

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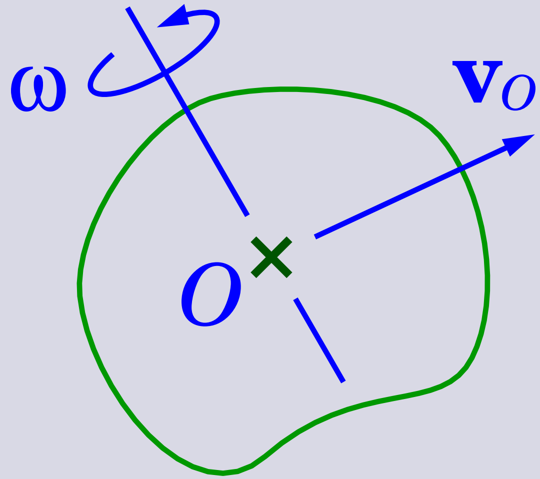
6D vectors are routinely expressed in Plücker coordinates;

but a coordinate system on a vector space is defined by a basis;

so

1. what are the basis vectors for Plücker coordinates?
2. why should we want to know?

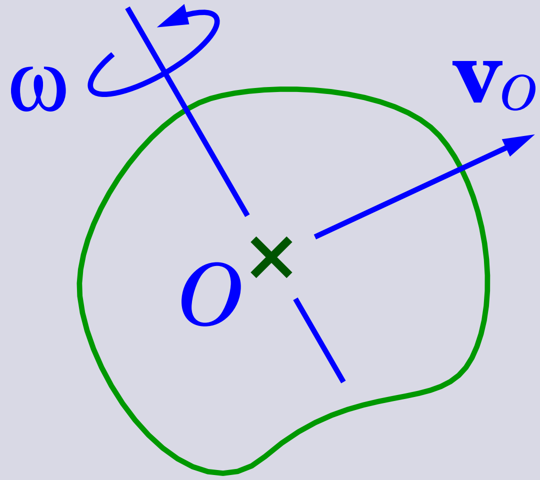
Rigid Body Velocity



The velocity of a rigid body is specified by

1. choosing a point, O, anywhere in space
2. specifying the linear velocity, $\mathbf{v}_O$, of the point in the body that coincides with O
3. specifying the angular velocity, $\boldsymbol{\omega}$, of the body as a whole

Rigid Body Velocity



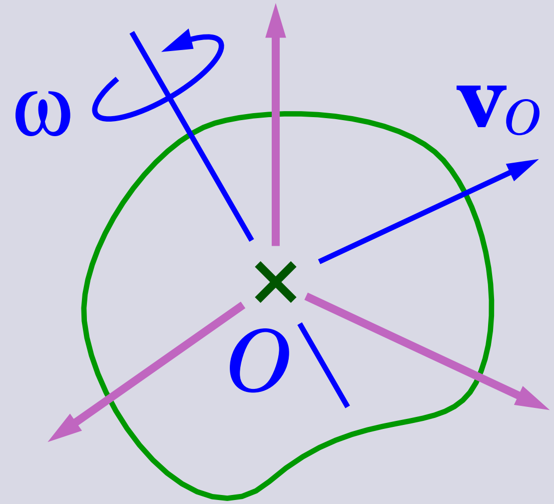
The body is then deemed to be

translating with a linear velocity of $\mathbf{v}_O$

while simultaneously

rotating with an angular velocity of ω
about an axis passing through O

Add a Coordinate Frame

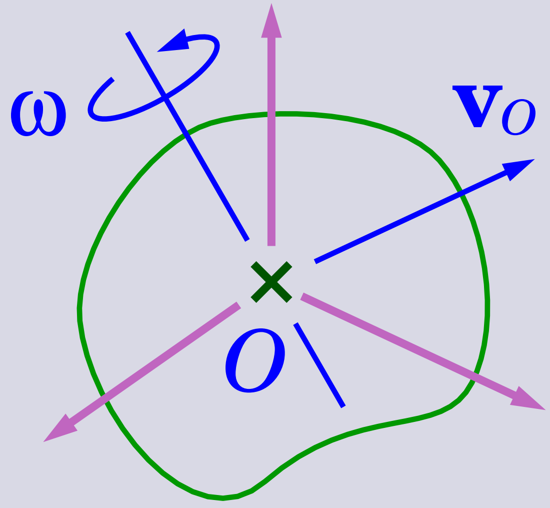


$\boldsymbol{\omega}$ and $\mathbf{v}_O$ can now be expressed in Cartesian coordinates:

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad \text{representing} \quad \omega_x\,\mathbf{i} + \omega_y\,\mathbf{j} + \omega_z\,\mathbf{k}$$

$$\begin{bmatrix} v_{Ox} \\ v_{Oy} \\ v_{Oz} \end{bmatrix} \quad \text{representing} \quad v_{Ox}\,\mathbf{i} + v_{Oy}\,\mathbf{j} + v_{Oz}\,\mathbf{k}$$

and the 6D velocity vector, $\hat{\mathbf{v}}$, can be expressed in Plücker coordinates as:



$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \\ v_{Ox} \\ v_{Oy} \\ v_{Oz} \end{bmatrix}$$

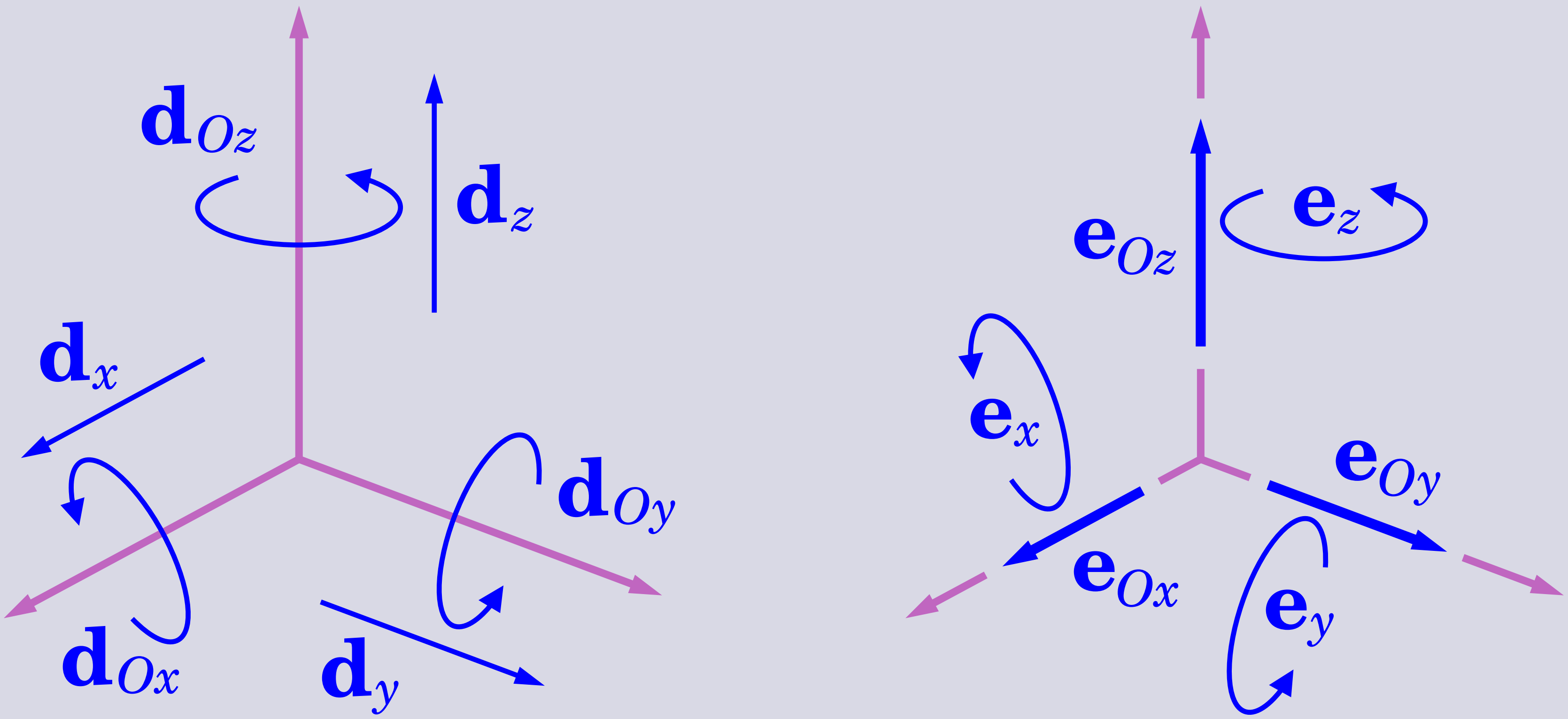
representing

$$\hat{\mathbf{v}} = \omega_x \, \mathbf{d}_{Ox} + \omega_y \, \mathbf{d}_{Oy} + \omega_z \, \mathbf{d}_{Oz} + v_{Ox} \, \mathbf{d}_x + v_{Oy} \, \mathbf{d}_y + v_{Oz} \, \mathbf{d}_z$$

Plücker Basis Vectors

motion

force

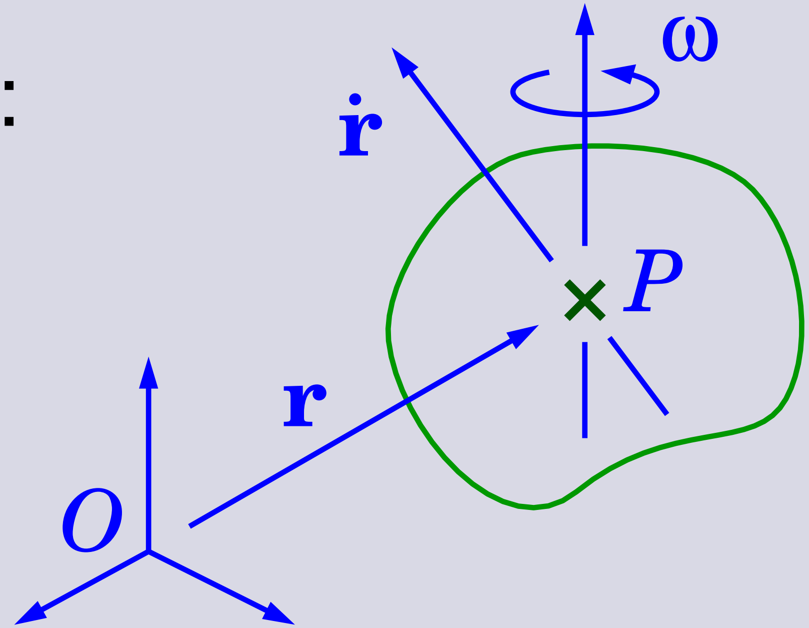


Plücker Basis Vectors:–

- define precisely the relationship between a Plücker coordinate vector and the quantity it represents
- clarify the concept of a 6D vector, and debunk some misconceptions
- plug a hole in our 6D vector theories
- provide a new analytical tool to users of 6D vectors

Example Misconception:

"The reduction point (P) is not the origin."



This mistake is the result of not realizing that the Plücker basis vectors are intrinsically tied to P. If P is moving, then the Plücker basis is changing with time.