Abstraction Heuristics in Planning

ICAPS 2008 —

Abstraction Heuristics in Planning

Patrik Haslum & Malte Helmert

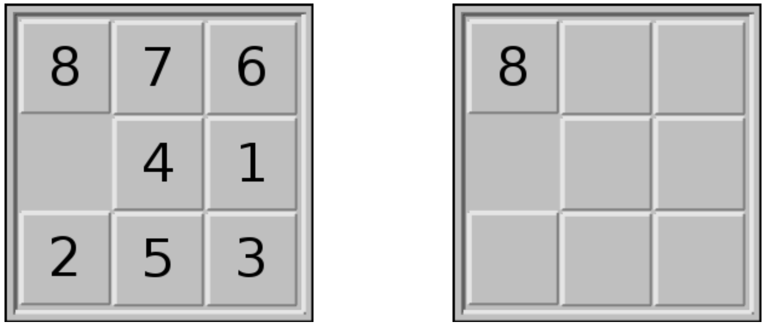
...from various places...

ICAPS 2008

Introduction

- Admissible heuristics are (often) defined as optimal solutions to a problem *relaxation*, that is easier to solve than the original problem.
- Here, we'll consider heuristics where:
  - The relaxed problem is an *abstraction* of the original.
  - The relaxed problem is *solved by search* (mostly).
- Canonical example: Pattern Databases

What's an Abstraction? A Classic Example



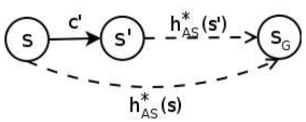
What's an Abstraction? A Formal Definition

- **Definition:** A *state space*, S, is a directed graph with labelled & weighted edges.
  - Vertices represent *states*.
  - Edges represent *state transitions*.
  - The edge label is an action/transition *name*.
  - The edge weight is the action/transition *cost*.
- A search problem consists of a state space, S, an *initial state*, s_I, and a set of *goal states* G.
  - Find a minimum-cost path through S from s_I to some $s_G \in G$, or prove that none exists.
- **Definition:** $h^*_S(s)$ denotes the cost of cheapest path through S from s to some $s_G \in G$ (∞ if no path exists).

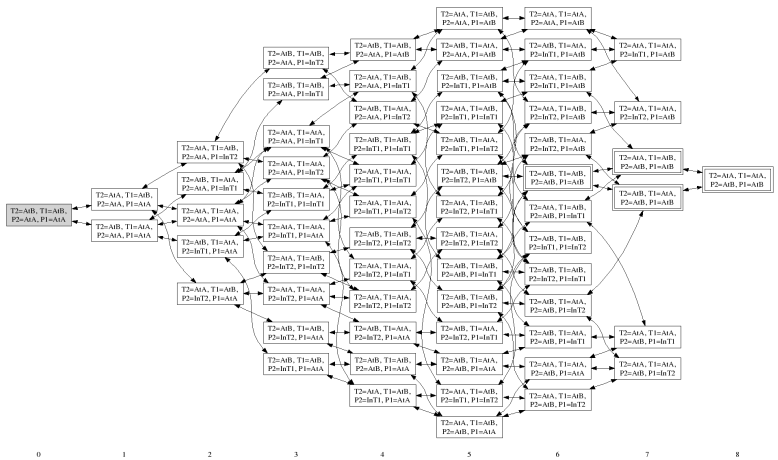
- **Definition:** An *abstraction* is a mapping, φ, from (states of) S to some *abstract space* AS, which *preserves labelled paths and goal states*.
  - If $s \xrightarrow{a:c} s'$ in S, then $\varphi(s) \xrightarrow{a:c'} \varphi(s')$ in AS, with $c' \leq c$.
  - If $s \in G_S$, then $\varphi(s) \in G_{AS}$.
- φ is a *homomorphism* iff AS has no transitions or goal states other than those required by the above.
- **Definition:** The corresponding *abstraction heuristic* is

$$h^\varphi(s) = h^*_{AS}(\varphi(s)).$$

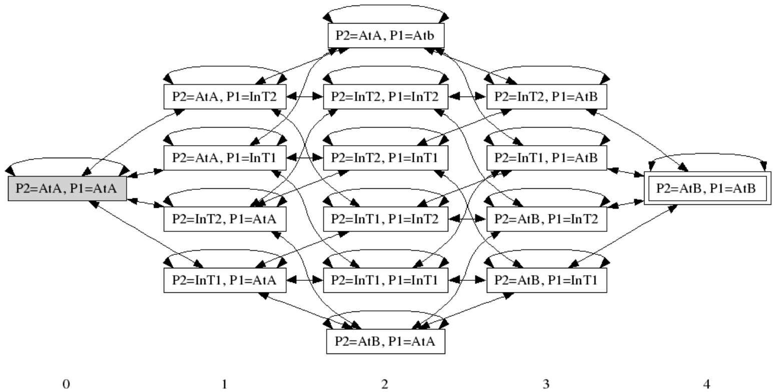
- **Theorem:** h^φ is admissible.
  Proof: Every s-to-goal path in S exists also in AS, and leads to a goal. Thus, *least* cost-to-goal from $\varphi(s)$ in AS can not be greater than in S.
- **Theorem:** h^φ is consistent/monotone.
  Proof: If $s \xrightarrow{a:c} s'$ in S, then $\varphi(s) \xrightarrow{a:c'} \varphi(s')$ in AS. Therefore $h^*_{AS}(s) \leq c' + h^*_{AS}(s')$, which implies $h^\varphi(s) \leq c' + h^\varphi(s') \leq c + h^\varphi(s')$.



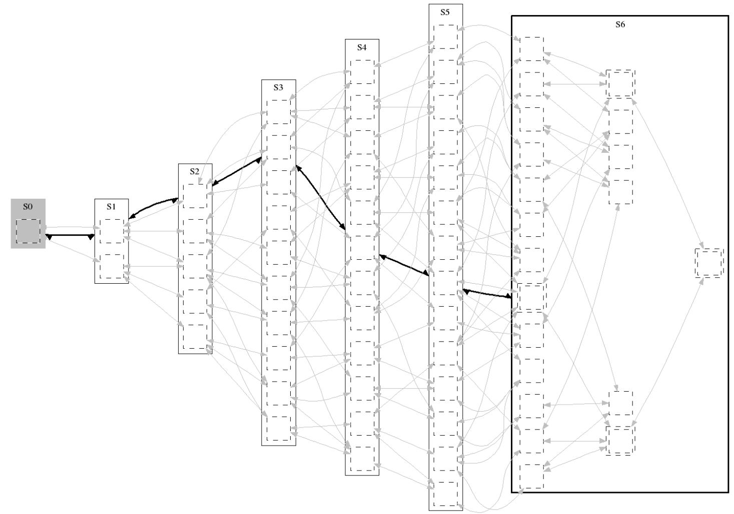
Example: Logistics (Original State Space)



Example: Logistics (Abstract State Space 1)



Example: Logistics (Abstract State Space 2)



Valtorta's Theorem (Generalised)

Theorem: Let S be a state space, s_I and G the initial and goal states, $AS = \varphi(S)$ an abstraction, $h^\varphi(s)$ the abstraction heuristic computed by blind search in AS.

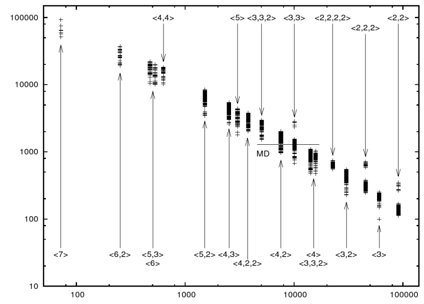
If an s_I–G-path in S is found by A* using h^φ as the heuristic, for any state s necessarily expanded by a blind search in S, either s is expanded by A* or $\varphi(s)$ is expanded while computing h^φ.

(Holte, Perez, Zimmer & MacDonald, 1995).

How to Beat It?

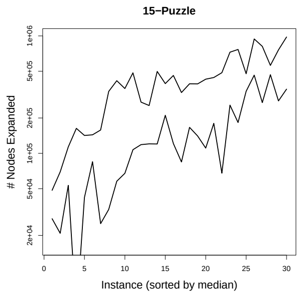
- Use *memory*: compute each h^*_{AS}-value only once.
  - Exhaustive reverse exploration, using $|AS|$ memory, only *expands* each state in AS once.
  - But memory is *limited*.
- Map *many* states in S *to one* in AS.
  - $|S|/|AS|$ h^φ-values for every computed h^*_{AS}-value, on average.
  - But the smaller $|AS|$, the less accurate is h^φ: $\overline{h^\varphi} \approx \log_b(|AS|)$.
- Don't use search to compute h^φ.
  - Choose φ so that $\varphi(S)$ has *structure* that can be exploited to compute $h^*_{\varphi(S)}$ more efficiently.

The $t \approx {}^n/_m$ Conjecture



Graph copied from Hernádvölgyi (2004).

Point: Average over 1000 instances.



Point: Max/min for 1 instance.

- Korf (1997) conjectured $t \approx {}^n/_m$, for IDA* search.
- Holds on average, but large variation among abstractions and across instances.

Abstraction Heuristics Applied to Planning

- In planning, we're given a *description*, in some *formal language* (STRIPS, PDDL, SAS+, *etc.*) of S.
  - Domain-independent: Can't assume any more about S than what must hold for any problem expressible in the input language.
  - Automatic: No human ingenuity should be required beyond writing the problem description.
- Focus on solving *one instance* (in domain-specific search, often many instances with same state space & goal).
  - Need to consider also precomputation in heuristic cost-accuracy trade-off.

- Abstraction heuristics have attractive properties for domain-independent planning:
  - They're general: Abstractions exist for every planning domain/instance.
  - They're largely automatic: Once φ chosen, heuristic computation can be done by generic, automatic procedure.
- But applying them to planning also presents particular challenges:
  - Typically many possible abstractions: How to choose a *good* one – automatically?
  - Many planning domains generate search spaces different from those typically considered in domain-specific search
  - *E.g.*, counting problems *vs.* permutation problems.

Representations of Planning Problems

- Planning problem representations are normally *factored*:
  - States are assignments to set V of *state variables*.
  - Transitions defined by set of *actions*, each with an *applicability condition* and an *effect* on subset of V.
  - Goal states defined by condition on $V' \subseteq V$.
- We'll assume a SAS-style representation:
  - More compact (important for memory-based heuristics).
  - Clearer structure.
- Automatic STRIPS-to-SAS+ conversion is possible.
- *Cave*: STRIPS problem may have several SAS+ encodings, not all equally good.

The SAS+ Representation

Definition: A *SAS+ representation* of a planning problem consists of

- A set of *state variables*, V.
- For each $V_i \in V$, a *finite domain of values*, *dom*(V_i).
- A set of *actions*: each action a has:
  - A *unique name* (a).
  - A *precondition*, *pre*(a): a *partial assignment* to V (*i.e.* a conjunction of assignments "*variable* = *value*", for some subset of variables).
  - An *effect*, *eff*(a): also a partial assignment.
  - A cost, *cost*(a).
- An *initial state*, I: a complete assignment to V.
- A *goal*, G: a partial assignment.

Definition: The representation *induces* a *search space*:

- States assign a value in *dom*(V_i) to V_i, for each $V_i \in V$.
  - "$s[V_i]$": the value of V_i in state s.
- Transitions induced by actions: $s \xrightarrow{a:\mathit{cost}(a)} s'$ iff
  - $s[V_i] = \mathit{pre}(a)[V_i]$ for all V_i mentioned in *pre*(a).
  - $s'[V_i] = \mathit{eff}(a)[V_i]$ for all V_i mentioned in *eff*(a).
  - $s'[V_i] = s[V_i]$ for all other V_i.
- Initial state: The (unique) state satisfying I.
- Goal: Set of states satisfying G.

Abstracting Transformations on Planning Problems

- Ignore one or more state variables.
  - *E.g.*, ignoring `T1` & `T2` yields abstract state space 1.
- Merge values in domain of a variable.
  - *E.g.*, merge `InT1` and `InT2` in *dom*(`P1`).
  - `unload P1 from T2 at` L
    `pre: P1=InT1-or-InT2, T2=At`L
    `eff: P1=At`L

SAS Encoding

```
P1:  {AtA, AtB, InT1, InT2}
P2:  {AtA, AtB, InT1, InT2}
T1:  {AtA, AtB}
T2:  {AtA, AtB}

load Pi in Tj at L
pre:  Pi=AtL, Tj=AtL
eff:  Pi=InTj

unload Pi in Tj at L
pre:  Pi=InTj, Tj=AtL
eff:  Pi=AtL

drive Tj L -> L'
pre:  Tj=AtL
eff:  Tj=AtL'
```


- Replace individual variables by counters.
  - *E.g.*, replace `P1` and `P2` with `#{Pi=AtA}`, `#{Pi=AtB}`, `#{Pi=InT1}` & `#{Pi=InT2}`.
  - `unload Pi from T2 at` L
    `pre: T2=At`L
    `eff: #{Pi=InT2}-=1, #{Pi=At`L`}+=1`
- Reduce the cost of one or more actions.
  - *E.g*, set cost of `drive` to zero.
  - Abstract state space same as original, but has cheaper paths.

SAS Encoding

```
P1:  {AtA, AtB, InT1, InT2}
P2:  {AtA, AtB, InT1, InT2}
T1:  {AtA, AtB}
T2:  {AtA, AtB}

load Pi in Tj at L
pre:  Pi=AtL, Tj=AtL
eff:  Pi=InTj

unload Pi in Tj at L
pre:  Pi=InTj, Tj=AtL
eff:  Pi=AtL

drive Tj L -> L'
pre:  Tj=AtL
eff:  Tj=AtL'
```

A Brief (and Very Selective) History of Abstraction Heuristics

- Prieditis (1993)
  - Catalog of abstracting problem transformations.
  - Automatic search for "abstractions that can be sped up".
- Culberson & Schaeffer (1996, 1998)
  - Pattern Database heuristic for the 15-Puzzle.
- Korf & Taylor (1996)
  Korf (1997)
  - Reconstruction of Manhattan distance as PDB & extension to dynamic additive 2-tile PDBs.
  - The $t \approx n/m$ conjecture.

- Holte & Hernádvölgyi (1999, 2000)
  Hernádvölgyi (PhD 2004)
  - Applied PDBs to several search problems.
  - Experimental test/validation of theory, including the $t \approx n/m$ conjecture.
  - Pattern selection by local search for SOP.
- Holte *et al.* (1995, 2005)
  - Hierarchical A* & IDA*: recursive on-the-fly search in abstract state spaces.
- Edelkamp (2001, 2002)
  - Applied PDBs to planning, using multi-valued state variable ("SAS+") representation.
  - Sufficient condition for additivity.
  - Symbolic representation of PDBs using BDDs.

- Haslum, Bonet & Geffner (2005)
  - Constrained abstraction.
  - Conflict-directed pattern selection.
- Edelkamp (2006)
  Haslum, Bonet, Helmert, Botea & Koenig (2007)
  - Pattern selection by local search for planning.
- Dräger, Finkbeiner, Podelski (2006)
  Helmert, Hoffman, Haslum (2007)
  - Automatic construction of general explicit-state abstractions.
- Katz & Domshlak (2008)
  - Structural pattern heuristics.

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Pattern Database Heuristics

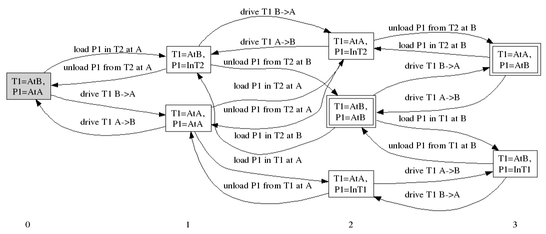
- Pattern databases (PDBs) are *memory-based* abstraction heuristics, in which the abstraction is typically a *projection*.
  - h^*_{AS} precomputed and stored for every abstract state.
  - $h^\varphi(s)$ computed by looking up the value for $\varphi(s)$.
- Let $V' \subset V$ be a subset of state variables: φ is a *projection on* V' iff $\varphi(s) = \varphi(s')$ iff s and s' agree on the value of every variable in V'.
  - Equivalent to *ignoring* variables *not in* V'.
  - Projecting transformation: Remove conditions & effects on variables not in V' from actions & goal.
  - *N.B.* This is an *over-approximation* (φ not necessarily a homomorphism).
- The kept variable set, V', is called the *pattern*.

- How to store h^*_{AS} & compute h^φ?
  - Map every abstract state $s \in AS$ to a unique index (*perfect hash function*).
  - *w.l.o.g.* *dom*$(V_i) = \{0, \ldots, |$*dom*$(V_i)| - 1\}$: variable-value assignment is a number in an "uneven" base.

    E.g., $V' = \{V_2, V_5\}$: *index*$(s) = (s(V_2) \cdot |$*dom*$(V_5)|) + s(V_5)$.
  - Store h^*_{AS} values in table, indexed by *index*(s).
  - More compact storage possible for certain state spaces (*e.g.*, permutations).
- How to compute h^*_{AS}?
  - Exhaustive, "cost-first" search *in reverse* through AS.
  - h^φ for forward search: backwards from goal states in AS.
  - h^φ for regression search: forwards from initial state in AS.

Example: Logistics

- 2 Packages (P1,P2), 2 Trucks (T1, T2), 2 locations (A and B).
- Projection on {T1,P1} (abstract space):



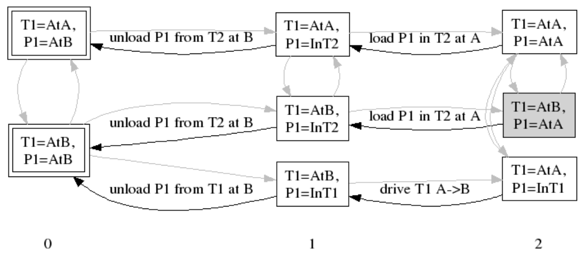
SAS Encoding

P1: {AtA, AtB, InT1, InT2}
P2: {AtA, AtB, InT1, InT2}
T1: {AtA, AtB}
T2: {AtA, AtB}

load Pi in Tj at L
pre: Pi=AtL, Tj=AtL
eff: Pi=InTj

unload Pi in Tj at L
pre: Pi=InTj, Tj=AtL
eff: Pi=AtL

drive Tj L -> L'
pre: Tj=AtL
eff: Tj=AtL'

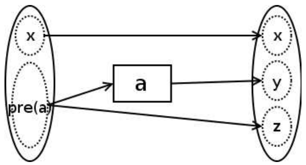


Progression PDB

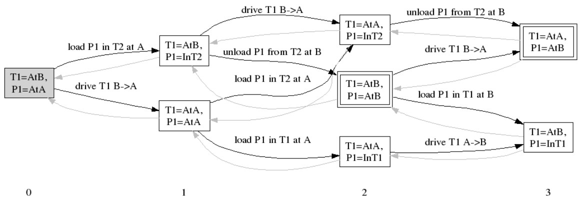
index(s): s	$h(s)$	*index*(s): s	$h(s)$
0: T1=AtA, P1=AtA	2	4: T1=AtB, P1=AtA	2
1: T1=AtA, P1=AtB	0	5: T1=AtB, P1=AtB	0
2: T1=AtA, P1=InT1	2	6: T1=AtB, P1=InT1	1
3: T1=AtA, P1=InT2	1	7: T1=AtB, P1=InT2	1

A Note on PDBs for Regression

- In regression, a search state is a condition to achieve.
  - Conditions may be *partial*.
  - Need to extend *dom*(V) with a "don't care" value $(*)$.
- "Regression in reverse" $\neq$ Forward action application!
- regress$(c, a) = (c - \textit{eff}(a)) \cup \textit{pre}(a)$
  if *eff*(a) contributes part of c and *eff*(a) doesn't contradict c.
- In reverse: $(\textit{pre}(a) \cup x) \xrightarrow{a} (x \cup y \cup z)$,
  where $(x \cup y \cap z) \cap \textit{eff}(a) = \emptyset$, $y \subseteq \textit{eff}(a)$, $y \neq \emptyset$, $z \subseteq \textit{pre}(a)$,
  $x \cap \textit{pre}(a) = \emptyset$, $x \cap y = y \cap z = x \cap z = \emptyset$.



- Another way to compute a regression PDB:
  - For *complete* $s \in AS$, compute $h^*_{AS}(s)$ (distance from $\varphi(s_I)$) by standard forward exploration.
  - For *partial* $s \in AS$, $h^*_{AS}(s) = \min_{s'} h^*_{AS}(s')$ over all *completions* s' of s.
- For *the kind of projection we've considered so far*, both methods yield same $h^\varphi(s)$ for all s.
- But, for *constrained projection*, this is not necessarily true – may yield an inconsistent heuristic.



Regression PDB

index(s): s	$h(s)$	*index*(s): s	$h(s)$
1: `T1=*, P1=AtA`	0	2: `T1=*, P1=AtB`	2
⋮		⋮	
6: `T1=AtA, P1=AtA`	1	11: `T1=AtB, P1=AtA`	0
7: `T1=AtA, P1=AtB`	3	12: `T1=AtB, P1=AtB`	2
⋮		⋮	

Combining PDB Heuristics

- If $A \subset B$, $h^A(s) \leq h^B(s)\ \forall s$ (h^B *dominates* h^A).
- $\max(h^A, h^B)$ is admissible & consistent.
- Patterns A and B are *additive* if no action has an effect on variables in both.
- If A and B are additive, $h^A + h^B$ is admissible.

  Proof: Least cost solution paths in $\varphi^A(S)$ and $\varphi^B(S)$ have no action in common.
- *N.B.* This is a *sufficient condition* only.
- $\max(h^A(s), h^B(s)) \leq h^A(s) + h^B(s) \leq h^{A \cup B}(s)$.
  - Additivity implies no "positive" interaction between abstract plans in $\varphi^A(S)$ and $\varphi^B(S)$, but there can still be "negative" interactions.

The Canonical Heuristic Function

- Let $\mathcal{C} = \{P_1, \ldots, P_m\}$ be a collection of patterns.
- The *canonical heuristic*, $h^{\mathcal{C}}$, is defined as

$$h^{\mathcal{C}}(s) = \max_{\mathcal{C}' \in \text{m.a.s.}(\mathcal{C})} \sum_{P \in \mathcal{C}'} h^P(s)$$

  where m.a.s.$(\mathcal{C})$ is the set of **m**aximal **a**dditive **s**ubsets of $\mathcal{C}$.
  - $|\text{m.a.s.}(\mathcal{C})|$ can be exponential in $|\mathcal{C}|$.
- *E.g.*, if $\mathcal{C} = \{\{\texttt{P1}, \texttt{T1}, \texttt{T2}\}, \{\texttt{P2}, \texttt{T1}, \texttt{T2}\}, \{\texttt{P1}\}, \{\texttt{P2}\}\}$,
  $h^{\mathcal{C}} = \max(h^{\{\texttt{P1,T1,T2}\}} + h^{\{\texttt{P2}\}}, h^{\{\texttt{P2,T1,T2}\}} + h^{\{\texttt{P1}\}}, h^{\{\texttt{P1}\}} + h^{\{\texttt{P2}\}})$
  ($h^{\{\texttt{P1}\}} + h^{\{\texttt{P2}\}}$ dominated by first two, can be left out).
- $h^{\mathcal{C}}$ is admissible & consistent, and *dominates every other combination* of h^P for $P \in \mathcal{C}$ under same condition for additivity.

Combining PDB Heuristics: A Special Case

- Consider the 15-Puzzle, and a pattern collection containing all pairs of tiles and single tiles:
  $\mathcal{C} = \{\{\texttt{T1}, \texttt{T2}\}, \{\texttt{T1}, \texttt{T3}\}, \ldots, \{\texttt{T14}\}, \{\texttt{T15}\}\}$.
  - 210 PDBs with 256 states, 15 PDBs with 16 states.
  - 225225 admissible sums (each of 7 pairs and 1 single).
- How efficiently find maximum admissible sum for state s?
  - Make a complete graph over tiles with edge $\texttt{T}i$–$\texttt{T}j$ weight equal to $h^{\{\texttt{T}i,\texttt{T}j\}}(s)$.
  - Solve weighted matching problem, in time $O(n^3)$.

    (Korf & Taylor 1996)
- Not specific to $(n^2 - 1)$-Puzzles.
  - Applicable to any set of mutually additive variables.
  - But tractable only for collection of *patterns of size 2* (hypergraph matching is intractable).

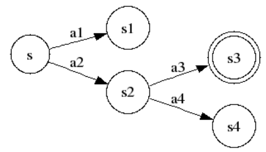
Generalised Additivity: Cost Distribution

- $h^{\varphi_1} + h^{\varphi_2}$ is admissible when no action may appear in optimal solution in both abstractions $\varphi_1(S)$ and $\varphi_2(S)$.
- 1st Generalisation: $h^{\varphi_1} + h^{\varphi_2}$ is admissible when no action *contributes to the cost* of optimal solution in both abstractions $\varphi_1(S)$ and $\varphi_2(S)$.
  - If every action that has an effect in both patterns A and B is given *zero cost* in either $\varphi^A(S)$ or $\varphi^B(S)$, $h^A + h^B$ is admissible.
- 2nd Generalisation: $h^{\varphi_1} + h^{\varphi_2}$ is admissible if for every action a, the *sum of its contributions* to the cost of optimal solution in $\varphi_1(S)$ and $\varphi_2(S)$ *does not exceed its cost in* S.
  - $\forall a$ $\mathit{cost}_{\varphi_1}(a) + \mathit{cost}_{\varphi_2}(a) \leq \mathit{cost}(a)$.

Optimal Cost Distribution over Abstractions

- Cost of reaching $s_G \in G$ from s in $\varphi(S)$ can be formulated as an LP, of size $O(|\varphi(S)|)$, in which action costs are variables.
- Heuristic value of s for the *optimal* cost distribution over abstractions $\varphi_1(S), \ldots, \varphi_k(S)$ can also be formulated as an LP: "union" of LPs for each $\varphi_i(S)$, plus admissibility constraint.

(Katz & Domshlak 2008)



$$
\begin{aligned}
&\max x(s) \\
\text{s.t. } & x(s) \leq y(a_1) + x(s_1) \\
& x(s) \leq y(a_2) + x(s_2) \\
& x(s_2) \leq y(a_3) + x(s_3) \\
& x(s_2) \leq y(a_4) + x(s_4) \\
& x(s_3) = 0 \\
& \ldots \\
& 0 \leq y(a_1) \leq \mathit{cost}(a_1) \\
& 0 \leq y(a_2) \leq \mathit{cost}(a_2) \\
& \ldots
\end{aligned}
$$

Constrained Projection

- The projecting transformation, "ignore variables not in P", can be *more relaxed* than the abstraction $\varphi^P(S)$.
  - Abstract state s' can be reachable in the transformed problem even if no s such that $\varphi^P(s) = s'$ reachable in S. ("*spurious states*").
  - Transition $s' \xrightarrow{a} t'$ can be allowed in the transformed problem even if for no $s \in S$ such that $\varphi(s) = s'$, *pre*(a) holds in s.
- This results in weaker PDB heuristics.
- Projection can be strengthened by *enforcing state constraints* valid in original space in the abstract space.

(Haslum, Bonet & Geffner 2005)

Example: 8-Puzzle

8	7	6
	4	1
2	5	3

Init

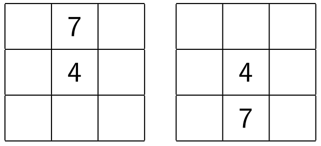
	1	2
3	4	5
6	7	8

Goal

SAS Encoding

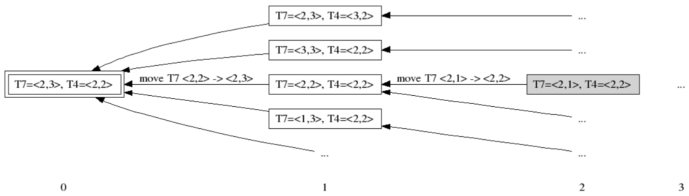
```
Ti:  {<1,1>, ..., <3,3>} (<col,row>)
Blank:  {<1,1>, ..., <3,3>}

move T1 <1,1> -> <1,2> (down)
pre:  T1=<1,1>, Blank=<1,2>
eff:  T1=<1,2>, Blank=<1,1>

move T1 <1,1> -> <2,1> (right)
pre:  T1=<1,1>, Blank=<2,1>
eff:  T1=<2,1>, Blank=<1,1>

...
```

- Projection onto Ti yields Manhattan distance for tile i.
- Abstractions are additive: sum over all but blank yields standard MD heuristic.



Init Goal

- Tiles 7 and 4 are in *linear conflict*.
- But projection onto {T4,T7} still yields only 2 – same as $h^{\{\mathtt{T4}\}} + h^{\{\mathtt{T7}\}}$.



- Short-cut through an "impossible" abstract state!

Solutions to the Problem?

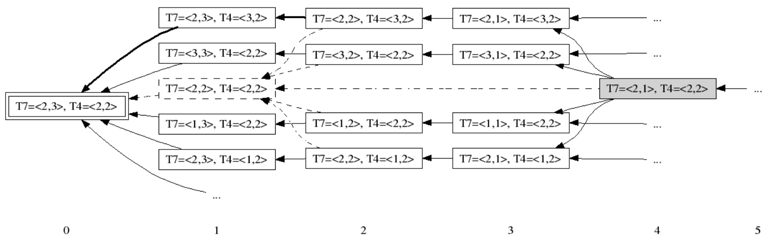
- Include `Blank` variable in abstraction:
  - Larger PDB.
  - Projection onto disjoint tile-sets not additive (unless `Blank`-only moves given zero cost).
- Encode the problem in a different way:

```
move T1 <1,1> -> <1,2>
pre:  T1=<1,1>, T2=/=<1,2>, T3=/=<1,2>, ...,
        T7=/=<1,2>, T8=/=<1,2>
eff:  T1=<1,2>
```

  - How to find right (re-)formulation *automatically*?
- Enforce *constraints* of original problem in abstract space.

Constrained Projection

- Let C be a collection of state invariants that hold in S.
  - *E.g.*, permanent mutexes ($\neg$T7=<2,2> $\vee$ $\neg$T4=<2,2>).
- In the constrained projection $\varphi^P_C(S)$, action a applies in state $\varphi^P_C(s)$ *iff pre*(a) and s agree on variables in P *and pre*$(a) \cup s$ does not violate any invariant in C.
- $h^{\varphi}_C(s) = h^*_{\varphi_C(S)}(s)$ is *admissible* and *consistent*.
  Proof: $\varphi_C(S)$ is a subgraph of $\varphi(S)$. Any edge in $\varphi(S)$ not in $\varphi_C(S)$ can not be part of any s_I-G-path in S (action not applicable, or applied in an unreachable state).

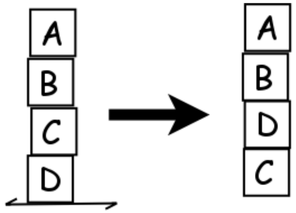


$$h^{\{\mathtt{T4,T7}\}}_C = 4$$

Pattern Selection for PDB Heuristics

- We can combine *any* collection of patterns, but not all yield equally *good* heuristics.
- More and/or larger PDBs are often better (and never worse) but memory is *limited* – how make best use of it?
  - Maxing several smaller PDBs often better than one large.
  - Exploit additivity: memory required for $h^{A \cup B}$ is *product* of that for h^A and h^B.
  - Recall: $\max(h^A(s), h^B(s)) \leq h^A(s) + h^B(s) \leq h^{A \cup B}(s)$.
- Require *automatic* (and not too costly) selection.
- Some approaches in planning literature:
  - Bin-packing.
  - Conflict-directed construction.
  - Local search.
- Problem representation may affect heuristic quality.

Example: Blocksworld (3op)

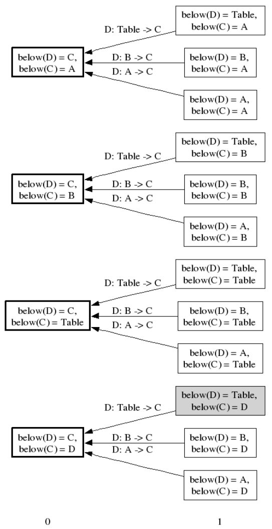


SAS Encoding 1:

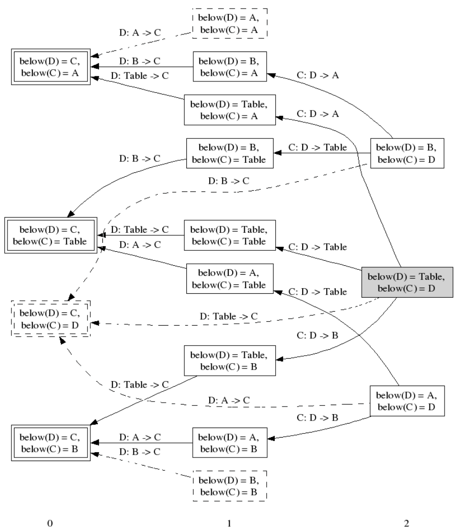
```
below(?X): {A, B, C, D, Table}
clear(?X): {true, false}

move ?X : ?X -> ?Z
pre:  below(?X)=?Y, clear(?X)=true, clear(?Z)=true
eff:  below(?X)=?Z, clear(?Y)=true, clear(?Z)=false

move ?X : ?Y -> Table
pre:  below(?X)=?Y, clear(?X)=true
eff:  below(?X)=Table, clear(?Y)=true

...
```

SAS Encoding 2:

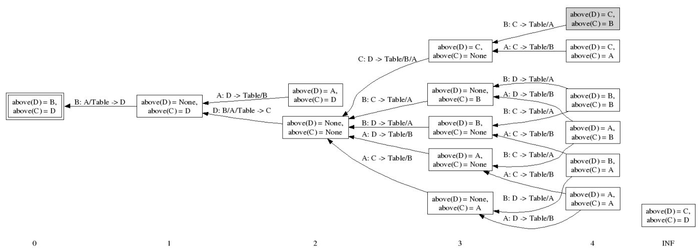
```
above(?X): {A, B, C, D, None}
ontable(?X): {true, false}

move ?X : ?X -> ?Z
pre:  above(?Y)=?X, above(?X)=None, above(?Z)=None
eff:  above(?Z)=?X, above(?Y)=None

move ?X : ?Y -> Table
pre:  above(?Y)=?X, above(?X)=None
eff:  above(?Y)=None, ontable(?X)=true

...
```

$h^{\{\texttt{below(C)},\texttt{below(D)}\}} = 1$

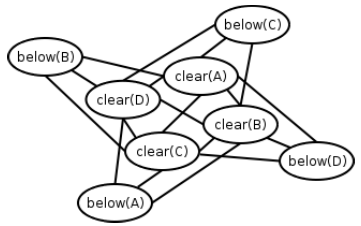


$h_C^{\{\texttt{below(C)},\texttt{below(D)}\}} = 2$

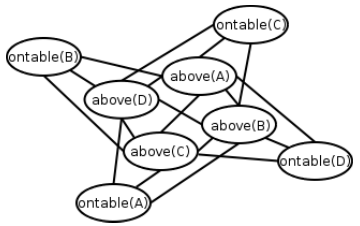


$h^{\{\texttt{above(C)},\texttt{above(D)}\}} = 4$

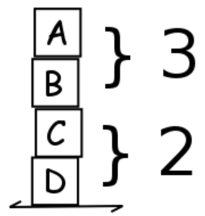
Encoding 1

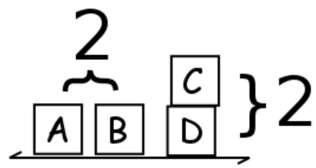


Encoding 2



- $h_C^{\{\texttt{below(C),below(D)}\}}$ and $h_C^{\{\texttt{below(A),below(B)}\}}$ are additive.
- $h^{\{\texttt{above(C),above(D)}\}}$ and $h^{\{\texttt{above(A),above(B)}\}}$ are not – can only take max.





$h_C^{\{\texttt{below(C),below(D)}\}} + h_C^{\{\texttt{below(A),below(B)}\}} = 5$

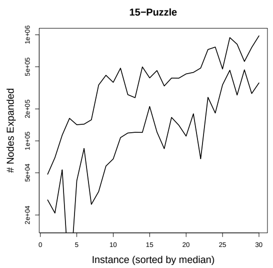
$\max(h^{\{\texttt{above(C),above(D)}\}}, h^{\{\texttt{above(A),above(B)}\}}) = 4$

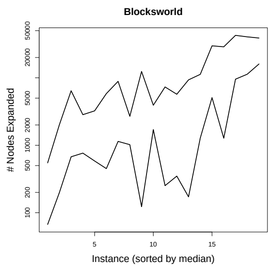
$h_C^{\{\texttt{below(C),below(D)}\}} + h_C^{\{\texttt{below(A),below(B)}\}} = 4$

$\max(h^{\{\texttt{above(C),above(D)}\}}, h^{\{\texttt{above(A),above(B)}\}}) = 3$

Bin-Packing Construction

- Given limit L on the size of any *single* PDB, partition $V_1, \ldots, V_n$ into *smallest number of patterns s.t.* $\prod_{V_j \in P_i} |dom(V_j)| \leq L$.
  - Bin-packing problem (if log-transformed).
  - NP-hard, but approximable.
- Ignores additivity.
- Ignores *relative usefulness* of placing V_i and V_j in same pattern.

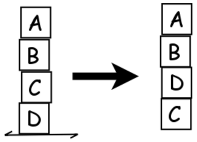




Conflict-Directed Construction

- For additive patterns A and B, when is $h^{A\cup B}$ more informative than $h^A + h^B$?
  - From a PDB, we can extract not only the cost of an optimal solution to the abstract problem, but also the *abstract plan*.
  - If there exists abstract plans for s in $\varphi^A(S)$ and $\varphi^B(S)$ that have *no conflict*, $h^{A\cup B}(s) = h^A(s) + h^B(s)$.
  - Conflict iff no interleaving of plans valid in $\varphi^{A\cup B}(S)$.
- Can't check *every* interleaving of *every pair* of abstract plans.
  - Pick *one* arbitrary abstract plan.
  - Ignore plan in $\varphi^B(S)$: check plan in $\varphi^A(S)$ for *dependencies* on / *mutexes* with some $V' \in B$.

Example: Blocksworld (3op)



{`below(A)`}: ()

{`below(B)`}: (`B:C->D`)

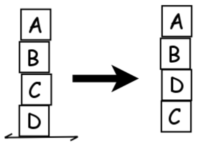
{`below(C)`}: ()

{`below(D)`}: (`D:Table->C`)

- Plan (`B:C->D`) conflicts with plan () for {`below(A)`}: prec. `clear(B)` mutex with `below(A)` = `B` always true.
- Plan (`B:C->D`) conflicts with plan () for {`below(C)`}: prec. `clear(D)` mutex with `below(C)` = `D` always true.
- Plan (`D:Table->C`) conflicts with plan () for {`below(C)`}: prec. `clear(D)` mutex with `below(C)` = `D` always true.

Conflict-Directed Construction

- Given additive variables, $V_1, \ldots, V_n$:
  - Initialise collection $\mathcal{C}$ with pattern $\{V_i\}$ for each V_i with goal value.
  - Compute PDB for each pattern in $\mathcal{C}$, analyse abstract plans for conflicts.
  - Merge patterns with most conflicts, if resulting pattern not too large (limit L on PDB size).
  - Repeat until no conflicts or no feasible mergers.



{`below(A)`, `below(B)`}:
(`A:B->Table`, `B:C->D`, `A:Table->B`)

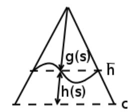
{`below(C)`}: ()

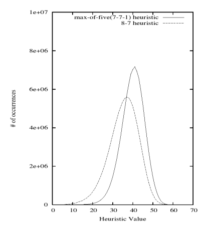
{`below(D)`}: (`D:Table->C`)

- Plan (`A:B->Table`, `B:C->D`, `A:Table->B`) conflicts with empty plan for {`below(C)`}: prec. `clear(D)` mutex with `below(C)` = `D` always true.
- Plan (`D:Table->C`) conflicts with plan () for {`below(C)`}: prec. `clear(D)` mutex with `below(C)` = `D` always true.

A Little Theory: How Useful is Heuristic h?

- $E_h(c)$: # nodes expanded by *tree search* to depth (cost bound) d with heuristic h; *i.e.*, # nodes with $g(s) + h(s) \leq d$.
- Blind search: $E_0(d) \approx b^d$.
- $E_h(d) \approx b^{d-\bar{h}}$
  - $\bar{h}$: expected value of $h(s)$.
  - But many more s with *high* $g(s)$ & *small* $h(s)$ encountered in search.
- $E_h(d) \approx \sum_{k=0,\ldots,d} N_{d-k} P_h(k)$
  - N_i: # nodes with acc. cost (g-value) i.
  - $P_h(k)$: probability that $h(s) \leq k$, for s drawn uniformly at random from the search tree. (Korf, Reid & Edelkamp 2001)





Graph copied from Holte, Felner, Newton, Meshulam & Furcy (2006).

Pattern Selection by Local Search

- Perform a (local) search in the *space of pattern collections*.
- How compare/rank collections? By *estimate* of $E_h(d)$.
- Instances of this scheme in planning literature:
  - Evolutionary algorithm over bounded-size collections, ranking by $\overline{h}$. (Edelkamp 2006).
  - Hill-climbing search over "growing" collections, ranking by estimated *reduction in search effort*. (Haslum, Bonet, Helmert, Botea & Koenig 2007).

Hill-Climbing Pattern Search

- Hill-climbing search.
- Initial pattern collection: $\mathcal{C} = \{\{V\} \mid V \text{ in goal}\}$.
- Neighbourhood of $\mathcal{C}$: $\mathcal{C}' = \mathcal{C} \cup \{P \cup \{V\}\}$, for any $P \in \mathcal{C}$ and $V \notin P$, unless $P \cup \{V\}$ or $\mathcal{C}'$ too large.
- Neighbourhood extends $\mathcal{C}$ with *one new pattern* P', which extends a pattern $P \in \mathcal{C}$ with *one new variable*.
- Neighbourhood ranking: by estimated reduction in search effort: $E_{h^{\mathcal{C}}} - E_{h^{\mathcal{C}'}}$.

- Estimating $E_{h^{\mathcal{C}}} - E_{h^{\mathcal{C}'}}$:
  - Recall: $E_{h^{\mathcal{C}}}(d) \approx \sum_{k=0,\ldots,d} N_{d-k} P_{h^{\mathcal{C}}}(k)$.
  - Effort saved by $h^{\mathcal{C}'}$ over $h^{\mathcal{C}}$: $\sum_k N_{d-k}(P_{h^{\mathcal{C}}}(k) - P_{h^{\mathcal{C}'}}(k))$
  - Estimate (m samples): $\frac{1}{m} \sum_{n_i} \sum_{h^{\mathcal{C}}(n_i) \leqslant k < h^{\mathcal{C}'}(n_i)} N_{d-k}$.
  - Simplified estimate: $\frac{1}{m}|\{n_i \mid h^{\mathcal{C}}(n_i) < h^{\mathcal{C}'}(n_i)\}| =$ *probability* that $h^{\mathcal{C}'}(n) > h^{\mathcal{C}}(n)$ for node n drawn uniformly at random from tree.
- Testing $h^{\mathcal{C}}(n_i) < h^{\mathcal{C}'}(n_i)$:
  - Sample states by random walk (to depth $\approx w \cdot h^{\mathcal{C}}(s_I)$).
  - Compute $h^{\mathcal{C}'}(s)$ for each sampled state – cost-effectively.
  - $\mathcal{C}'$ has only one new pattern $P' = P \cup \{V\}$: compute $h^{P'}(s)$ by search in abstract state space of P' using h^P as *heuristic*.

Additional Trix

- Consider patterns over *spanning* subset S of state variables
  - Assignment to variables in S plus mutex constraints determines values of all variables.
  - Select S to minimise $|S|$ and maximise additivity in S.
- Static: Add V to pattern P only if V causally connected to some variable in P – if not, $h^{P \cup \{V\}}$ is never better than $h^P + h^{\{V\}}$.
- Statistical: After each "epoch" of $\frac{m}{k}$ samples, calculate confidence interval $[l_i, u_i]$ for improvement of $\mathcal{C}'_i$: if $u_i < l_j$, $\mathcal{C}'_i$ is *very unlikely* to be better than $\mathcal{C}'_j$, so stop evaluating $\mathcal{C}'_i$.
- Stopping condition: No more patterns fit within size limits, or *improvement* of best new pattern *too small* (*e.g.*, $\leq 1\%$).

- No accurate *absolute* prediction of $E_{h^{C}} - E_{h^{C'}}$.
  - Simplified formula (disregards weight N_{d-k}).
  - Formula for $E_h(d)$ measures *tree search* effort – not necessarily same as *graph search* effort.

  – but sufficiently good measure of *relative merit* of PDB heuristics *in pattern search neighbourhood*.
- Neighbourhood evaluation is computationally expensive.
- Neighbourhood may not contain any improving pattern.
  - *E.g.*, Logistics with 2 Trucks: $h^{\{\mathtt{P1}\}}(s_I) = 2$, $h^{\{\mathtt{P1,T1}\}}(s_I) = 2$, $h^{\{\mathtt{P1,T2}\}}(s_I) = 2$, but $h^{\{\mathtt{P1,T1,T2}\}}(s_I) = 4$.
  - General problem with "disjunctive resources".

Symbolic Representations

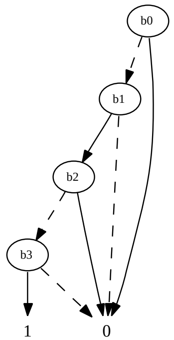
- "Symbolic" refers to use of decision diagrams to compactly represent *functions*.
- Binary Decision Diagrams (BDDs) represent *boolean* functions of *binary arguments*.
  - Can represent a *set of bit vectors*: $\alpha(b_1, \ldots, b_n) = 1$ iff $(b_1, \ldots, b_n) \in$ set.
  - Set operations ($\cup$, $\cap$, $= \emptyset$, *etc*) can be performed on (*ordered*) BDDs.
  - Key to efficient *set-based* search algorithms.
- Algebraic Decision Diagrams (ADDs) represent (partial) mappings from bit vectors to *arbitrary domain*.
- Decision diagrams *can* be much more *compact* than a tabular representation – but no guarantee.

Example: Logistics

- Encode state as bit vector by writing values in binary.

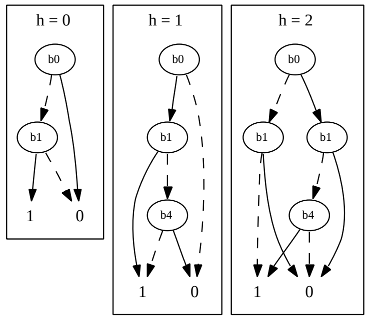
		P1		P2		T1	T2
Bit:		b0	b1	b2	b3	b4	b5
=	AtA	0	0	0	0	0	0
=	AtB	0	1	0	1	1	1
=	InT1	1	0	1	0		
=	InT2	1	1	1	1		

- Example BDD encoding set of goal states G.



How to Represent a PDB With a BDD

- Array of BDDs: $A[i]$ holds set of states with $h(s) = i$.



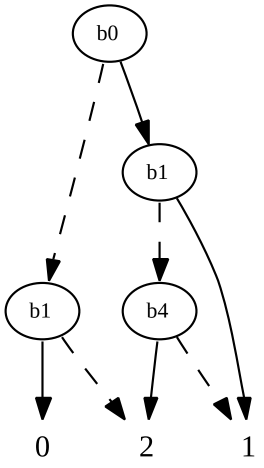
- Fast extraction of *set of states* with $h = i$ – important operation in some symbolic heuristic search algorithms.
- But finding $h(s)$ for a *single state* s requires linear scan.
- Direct construction by symbolic breadth-first search in abstract space.

(Edelkamp 2002, 2007)

- Alternative: Single BDD holding set of pairs $(s, h(s))$, with $h(s)$ encoded in binary.

How to Represent a PDB With an ADD

- Map states to PDB values.
- Fast look-up of $h(s)$ for a *single state* s.
- Probably most compact representation: up to 10^3 *times smaller* than corresponding PDB table in some planning domains. (Ball & Holte, 2008)
- But *not always* smaller.



Abstraction Heuristics in Planning

ICAPS 2008 —

Abstraction Heuristics in Planning

Malte Helmert & Patrik Haslum

...from various places...

ICAPS 2008

Limits on the Power of PDB Heuristics

- The power of PDB heuristics depends on *available memory*.
  - If memory not bounded, a PDB could contain all of S.
  - But time to compute the PDB also linear in PDB size.
- Consider search spaces of increasing size, S_n defined over variables $V_1, \ldots, V_n$, with $h^*(s_I) \approx O(|G|) \approx O(n)$.
  - *E.g.*, instances of growing size in a planning domain.
  - Plan length grows with size in all interesting domains.
- Examine the *asymptotic accuracy*, $h(s_I)/h^*(s_I)$ in the *limit of large* n.

- Requirement: PDB size (and computation time) is *polynomial in* n.
  - Consequence: Each pattern P can contain at most $O(log(n))$ variables.
  - $h^P(s_I)/h^*(s_I) \to 0$ as $n \to \infty$: h^P can be arbitrarily inaccurate.
- Does exploiting additivity help?
  - Can use $O(p(n))$ PDBs with $O(log(n))$ variables each.
  - There are planning domains where additive PDBs achieve (reasonably) good accuracy also for large n.
  - There are also planning domains where additive PDBs do *not* achieve better accuracy, *in the limit of large* n.

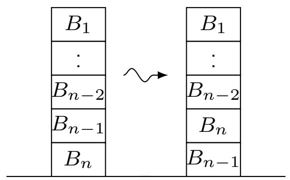
Example: Gripper

- Move n balls from `A` to `B`:
- $h^*(s_I) = 2n + n$ ($n \times$ `pick up` & `put down` $+ \, ^1\!/\!_2 n \times 2 \times$ `go`).
- $h^{\{\mathtt{B}i_1,\ldots,\mathtt{B}i_k\}}$ counts $k \times$ `pick up` & `put down`.
- $h^{\{\mathtt{B1}\}}(s_I) + \ldots + h^{\{\mathtt{B}n\}}(s_I) = 2n \geq \, ^2\!/\!_3 h^*(s_I)$.
- $h_C^{\{\mathtt{B}i_1,\ldots,\mathtt{B}i_k,\mathtt{Robot}\}}$ counts $k \times$ `pick up` & `put down` $+ \, ^1\!/\!_2 k \times 2 \times$ `go`.
- Additive only if at most one pattern includes `Robot`.
- At most $log(n)$ of n `go` counted by any sum: $h^{\mathcal{C}}(s_I) \to \, ^2\!/\!_3 n$ as $n \to \infty$.

SAS Encoding

```
Robot:  {AtA, AtB}
RGrip:  {empty, full}
LGrip:  {empty, full}
Bi:  {AtA, AtB, InR, InL}

pick up Bi with Right at X
pre:  Robot=AtX, Bi=AtX,
       RGrip=empty
eff:  Bi=InR, RGrip=full

...

put down Bi with Left at X
pre:  Robot=AtX, Bi=InL,
       LGrip=full
eff:  Bi=AtX, LGrip=empty

go X -> Y
pre:  Robot=AtX
eff:  Robot=AtY
```


Example: Blocksworld (3op)



- Swap blocks at base in tower of n blocks.
- $h^*(s_I) = 2n - 2$.

- Only `below(B`$_{n-2}$`)`, `below(B`$_{n-1}$`)` and `below(B`$_n$`)` differ between s_I and s_G (resp. only `above(B`$_{n-1}$`)` & `above(B`$_n$`)`).
- $h^{P_i}(s_I) > 0$ for at most three patterns P_i.
- Any sum $h^{P_{i_1}} + h^{P_{i_2}} + h^{P_{i_3}}$ can consider at most $3 \cdot log(n)$ variables, requiring no more than $6 \cdot log(n)$ moves: $h^{\mathcal{C}}(s_I)/h^*(s_I) \to 0$ as $n \to \infty$.

General Memory-Based Abstraction Heuristics

- To yield an *informative heuristic*, abstraction must *preserve relevant state distinctions*.
- To fit in memory, abstraction must have a *small representation*:
  - Can't have too many abstract states ($|\varphi(S)| \leq N$).
  - Succinct *encoding of abstraction mapping*, φ.
- PDBs can use a very compact encoding of the abstraction mapping ($|\mathsf{PDB}(P)| = O(|\varphi^P(S)|)$).
- But there may be *more informative* abstractions with N states that are *not projections*.
  - *E.g.*, Logistics, with 1 Package & m Trucks in 1 City: $h^{\{\mathtt{P1}\}} = 2 = h^{\{\mathtt{P1}\} \cup P}$, for *any* $P \subset \{\mathtt{T1}, \ldots, \mathtt{T}m\}$.
- Instead of preserving *a few* variables *perfectly*, it may be better to preserve *some information* about *all* variables.

- **Definition:** The *synchronised product* of two state spaces, S_1 and S_2, with same action sets, is denoted $S_1 \otimes S_2$ and defined as:
  - States in $AS_1 \otimes AS_2$: (s_1, s_2), *s.t.* $s_1 \in AS_1$, $s_2 \in AS_2$.
  - $(s_1, s_2) \xrightarrow{a} (s_1', s_2')$ in $AS_1 \otimes AS_2$ iff $s_1 \xrightarrow{a} s_1'$ in AS_1 and $s_2 \xrightarrow{a} s_2'$ in AS_2.
  - $(s_1, s_2) \in G_{AS_1 \otimes AS_2}$ iff $s_1 \in G_{AS_1}$ and $s_1 \in G_{AS_2}$.
- If $AS_1 = \varphi_1(S)$ and $AS_2 = \varphi_2(S)$ are abstractions of the same state space S, $AS_1 \otimes AS_2$ is also an abstraction of S which combines the information in both.
  - $\varphi_{1,2}(s) = (\varphi_1(s), \varphi_2(s))$.

- The state space S induced by a SAS+ problem over variables $V_1, \ldots, V_n$ can be reconstructed as

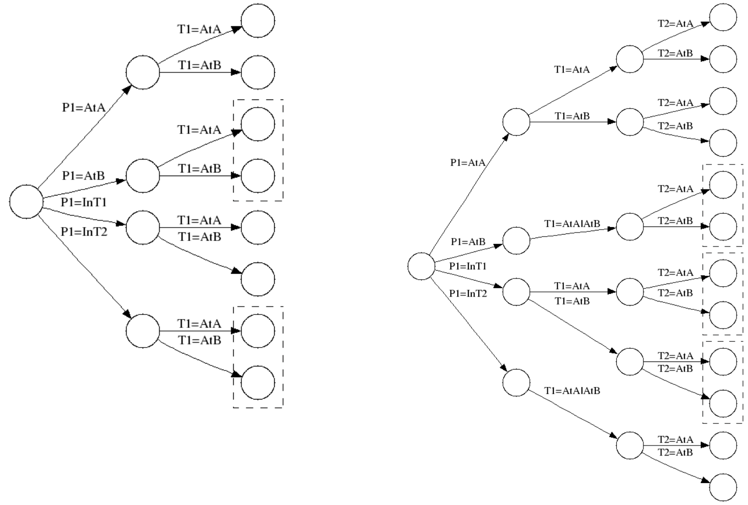
$$AS_{\{V_1\}} \otimes \ldots \otimes AS_{\{V_n\}},$$

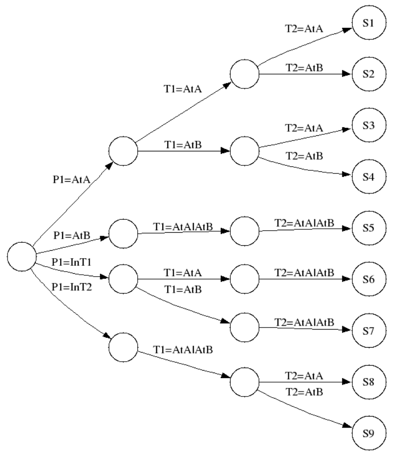
where $AS_{\{V_i\}} = \varphi^{\{V_i\}}(S)$ is the projection on $\{V_i\}$.
- If $\varphi(AS_{\{V_1\}} \otimes \ldots \otimes AS_{\{V_k\}})$ is an abstraction of $AS_{\{V_1\}} \otimes \ldots \otimes AS_{\{V_k\}}$, then

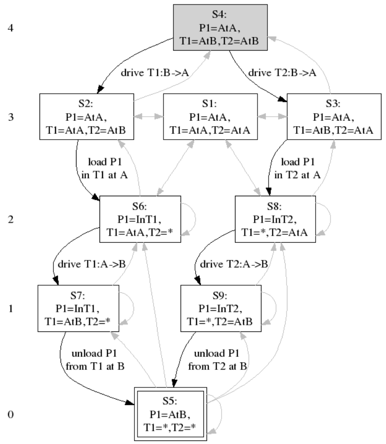
$$\varphi(AS_{\{V_1\}} \otimes \ldots \otimes AS_{\{V_k\}}) \otimes AS_{\{V_{k+1}\}} \otimes \ldots \otimes AS_{\{V_n\}}$$

is also an abstraction of S.
- Too large intermediate abstractions can be shrunk by *explicitly merging* some abstract states.

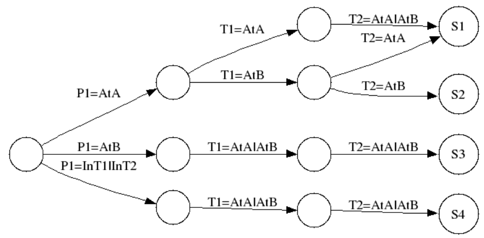
Example: Logistics (A Perfect Abstraction)

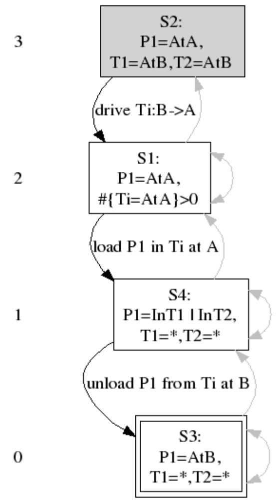






Example: Logistics (A Less Perfect Abstraction)





- But still better than $h^{\{P1\}}$!
- And better than $h^{\{P1,T1\}}$ and $h^{\{P1,T2\}}$.

Constructing General Abstractions

Algorithm for Constructing a General Abstraction

abs := all atomic projections $AS_{\{V_i\}}$ $(V_i \in V)$.
while abs contains more than one abstraction:
    select AS_1, AS_2 **from** abs
    shrink AS_1 and/or AS_2 until $|AS_1 \otimes AS_2| \leq N$
    abs := abs $\setminus \{AS_1, AS_2\} \cup \{AS_1 \otimes AS_2\}$
return the remaining abstraction

- Algorithm schema, to be instantiated with:
  - A *strategy* for selecting which pair of abstractions in the current pool to *merge*.
  - A *strategy* for how to *shrink* an abstraction.
  - A size bound N.
- Crucial to have *good* merging and shrinking strategies.

Merging Strategy

- Linear merging strategy: In each iteration after the first, choose the abstraction computed in the previous iteration as AS_1 and an atomic projection as AS_2.
  - Strategy defined by an ordering of atomic projections.
  - Maintains only one complex (non-atomic) abstraction.
  - Size of abstraction mapping (in "decision graph" form) bounded by $|V| \cdot N$.
- How to order atomic projections?
  - Start with a goal variable.
  - Add variables that appear in preconditions of operators affecting previous variables.
  - If that is not possible, add a goal variable.

Shrinking Strategy

- Construct $AS' = \varphi(AS)$ by selecting pairs of abstract states in AS to "collapse" (*i.e.*, map to same in AS').
- The mapping φ / the selection strategy is
  - *h-preserving* if it only collapses abstract states with equal distance-to-goal (h-value);
  - *g-preserving* if it only collapses abstract states with equal distance-from-init (g-value);
  - *f-preserving* if it is h- and g-preserving.
- If φ is h-preserving, $h^*_{AS}(s) = h^*_{AS'}(s)$ for all s – no loss of heuristic value.
- But some information relevant to variables merged in later may be lost.

- Shrink AS_1 until $|AS_1| \cdot |AS_2| \leq N$.
  - Never represents an abstraction with more than N states.
  - May preserve unimportant state distinctions in AS_2 at the expense of important state distinctions in AS_1.
- Use f-preserving abstraction as long as possible.
- If can't be f-preserving ($\#$ f-values $> N/|AS_2|$), prefer merging states with *high f-values* and *small f-difference*.
  - States with high f-values less likely to be explored.
- Tie-breaking: Prefer to preserve distinctions between states with small h-values.

Representational Power of General (Linear-Merge) Abstractions

- At least as powerful as PDBs.
  - Projection on $P \subset V$ is a special case.
  - $O(|V| \cdot N)$ overhead for general mapping representation.
- Captures additivity.
  - If P_1 and P_2 are additive patterns, for *any* h-preserving abstraction $AS_1 = \varphi_1(AS_{P_1})$ and $AS_2 = \varphi_2(AS_{P_2})$, the heuristic for $AS_1 \otimes AS_2$ dominates $h^{P_1} + h^{P_2}$.
  - But $|AS_1 \otimes AS_2|$ may be $O(|AS_{P_1}| \cdot |AS_{P_2}|)$.
  - If $\forall a\ \mathit{cost}(a) = 1$, there is an h-preserving abstraction φ such that $|\varphi(AS_1 \otimes AS_2)| = O(|AS_{P_1}| + |AS_{P_2}|)$, but it may not be linear-merge constructible.

Representational Power of General (Linear-Merge) Abstractions

- Can construct *perfect heuristics* with *polynomial-size abstractions* in some planning domains:
  - Gripper
  - Schedule
  - Two PROMELA variants.
- PDBs have unbounded error in these domains.

Example: Gripper

- $2^{n-1} \cdot (n^2 + n + 4)$ reachable states.
- $(3n - 3) \cdot 2$ state "classes" characterised by (#Bi=A, #Bi=B, #Bi=InX, Robot) – all states in each class have same distance-to-goal.
- Linear construction:
  - Add Robot, RGrip & LGrip, without abstraction.
  - Add each Bi in turn, merging abstract states that agree on number of balls in A, B and in grippers.

SAS Encoding

```
Robot:  {AtA, AtB}
RGrip:  {empty, full}
LGrip:  {empty, full}
Bi:  {AtA, AtB, InR, InL}

pick up Bi with Right at X
pre:  Robot=AtX, Bi=AtX,
       RGrip=empty
eff:  Bi=InR, RGrip=full

...

put down Bi with Left at X
pre:  Robot=AtX, Bi=InL,
       LGrip=full
eff:  Bi=AtX, LGrip=empty

go X -> Y
pre:  Robot=AtX
eff:  Robot=AtY
```


Structural Abstraction Heuristics

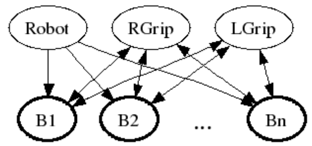
- PDBs & memory-based abstraction heurstics in general rely on *blind search* to compute h^*_{AS}: efficient *only if* the *size* of the abstract space is *small*.
- Alternative: Choose φ so that $\varphi(S)$ has a *structure* that permits computing h^*_{AS} by some *more effective method* than search.
  - *E.g.*, choose φ so that $\varphi(S)$ falls into a known class of *tractable optimal planning* problems.
  - h^{φ} still has all properties of abstraction heuristics: admissibility, monotonicity, condition for additivity, *etc.*

- But there aren't many known tractable optimal planning classes.
- Some examples:
  - SAS+-IAO (if actions have unit cost). (Jonsson & Bäckström 1998)
  - SAS+-UB with polytree causal graph of bounded in-degree. (Brafman & Domshlak 2003)
  - SAS+ with "fork" and "inverted fork" causal graphs, under various additional restrictions. (Katz & Domshlak 2008)
- These problem classes are *severely restricted*: Do any *usefull* abstractions fall in them?

Quick Reminder: The Causal and Domain Transition Graphs

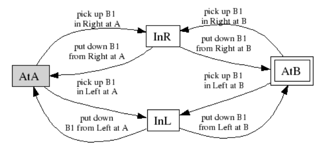
Definition:

- The *causal graph* is a graph over the *variables* of a SAS+ problem.
- Edge from V_i to V_j iff there exists an action a with an effect on V_j and precondition or effect on V_i.



Definition:

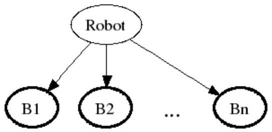
- The *domain transition graph* (DTG) is a graph over the *values* of *one variable* V.
- Edge from x to y iff there exists an action a with $V = x$ in *pre*(a) and $V = y$ in *eff*(a).



Tractable Classes With Fork-Shaped Causal Graphs

- Cost-optimal planning is tractable if
  1 The causal graph is a *fork*; and
  2(a) $|dom(V_r)| = 2$ for the root variable; or
  2(b) $|dom(V_i)|$ is bounded by a constant for every non-root variable.
- Projections inducing fork-shaped causal graphs found in many planning domains (n variables depending on 1).
- Domain-size restrictions can be achieved by *merging values*.

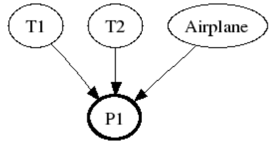
Projection on $\{\texttt{B1}, \ldots, \texttt{B}n, \texttt{Robot}\}$



Tractable Classes With "Inverse Fork"-Shaped Causal Graphs

- Cost-optimal planning is tractable if
  1 The causal graph is an *inverted fork*; and
  2 the domain of the (single) leaf variable V_l is bounded by a constant; and
  3 each action affecting V_l has a precondition on *at most one* other variable.
- Projections inducing inverse fork-shaped causal graphs also found in many planning domains (1 variable depending on n).

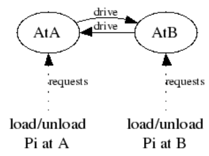
Projection on $\{\texttt{P1}, \texttt{T1}, \texttt{T2}, \texttt{Airplane}\}$



The SAS+-IAO Class

- A SAS+ planning problem is
  - *Interference-safe* (**I**) iff every non-unary action a is *irreplaceable* (removing the edge associated with a splits the DTG of every variable affected by a in separate components).
  - *Acyclic w.r.t. requestable values* (**A**) iff for every variable V_i, and for the set of *requestable values* $R \subset dom(V_i)$ (values that appear in $eff(a)$ for non-unary a or in $pre(a)$ for a not affecting V_i), the transitive closure of its DTG restricted to R is acyclic.
  - *Prevail-order preserving* (**O**) iff for every variable V_i and $x, y \in dom(V_i)$, $X \subset R(V_i)$, the shortest path from x to y passing each value in X also has minimal (subsequence-wise) conditions on other variables.

- Minimal-length planning for SAS+-IAO is tractable.
- *N.B.* The SAS+-IAO class contains problems with *arbitrary causal graphs*.
- The projection on *a single variable* satisfies the IAO restrictions (by definition).
- But there doesn't seem to be any non-trivial larger projections that do so in standard planning domains.
  - Many domains are *symmetric*: fail acyclicity and/or interference-safety restrictions.
  - Many domains have *alternative DTG paths*: fail interference-safety and/or prevail-order restrictions.



Hierarchical Abstraction Search

- PDBs *precompute* $h^*_{AS}(s)$ for *every* abstract state s.
- In solving a single problem, typically only a *small fraction* of PDB entries are used (may be $< 0.1\%$).
- Alternative: Compute $h^*_{AS}(\varphi(s))$ on "need to know basis", *i.e.*, only when $h^{\varphi}(s)$ evaluated.
  - Still using search to compute $h^*_{AS}(\varphi(s))$.
  - But not using *blind search*: a *hierarchy* of abstractions provides heuristics for search in AS.
  - Still using memory to avoid *recomputing* $h^*_{AS}(\varphi(s))$.

Caching in Hierarchical Abstraction Search

- Abstraction hierarchy: $AS_1 = \varphi_1(S)$, $AS_2 = \varphi_2(AS_1)$, ...
- What to store?
  - When an s–G-path in AS_i is found, we know $h^*_{AS_i}$ for *every state on this path*.
  - When an s–G-path in AS_i is found, we know that $h^*_{AS_i}(s') \geq h^*_{AS_i}(s) - g(s')$ for every s' explored in that search.
- How to use it?
  - Use stored $h^*_{AS_i}(s)$ whenever evaluating $h^{\varphi_i}(s')$ for $s' \in AS_{i-1}$ such that $\varphi_i(s') = s$.
  - Use stored lower bounds on $h^*_{AS_i}$ to focus search in AS_i.
  - When search in AS_i reaches s such that $h^*_{AS_i}(s)$ is known, short-cut the search.

How Effective Is It?

- The *total size* of hierarchy of abstract spaces, $|AS_1| + \ldots + |AS_n|$ may be much larger than $|S|$.
- With above caching strategies and suitable abstractions:
  - Hierarcical A* beats blind search in S in $\geq 50\%$ of instances across several domains.
  - But *never* in 100% of instances.

    (Holte, Perez, Zimmer & MacDonald, 1995).
- PDB computation searches *only* AS_1, but does so *exhaustively*.

Summary & Conclusions

- We've talked mostly about *memory-based abstraction heuristics*:
  - Using *projections* (PDBs).
  - Using more general explicit-state abstractions.
- These heuristics have been shown to be *very effective* in many domain-specific search applications.
- And they can be effective for *domain-independent planning* too, in spite of the fact that
  - we must include heuristic *construction* (*precomputation*) *time* in cost-benefit trade-off; and
  - we need *automatic* and *domain-independent* methods for *selecting good abstractions*.

Some Open Questions

- For memory-based abstraction heuristics,
  - we don't know how to trade off precomputation time against the value of heuristic information;
  - the pattern selection problem is *not solved*.
- There are alternatives to memory-based abstraction heuristics:
  - Hierarchical abstraction search
    – search abstract spaces only when needed.
  - Structural pattern heuristics
    – solve abstract problem by better method than search.
- But these have not been much explored in planning.
  - We don't know which is most the effective way, for which classes of problems.

Some Things We Haven't Talked About

- Many improvements to PDB heuristics in domain-specific search:
  - Dual look-up & exploiting problem symmetries.

    (Felner, Zahavi, Schaeffer & Holte 2005)
  - Storing "increment over base heuristic" in PDB.
  - Compressed PDBs. (Felner, Korf, Meshulam & Holte 2007)
  - Combining PDBs with perimeter search.

    (Linares Lopéz 2008)
- Non-heuristic uses of abstraction:
  - Problem simplification, safe abstraction & heirarchical planning.
  - The use of abstractions to *prove unreachability*, common in model checking.