A Qualitative Representation of Route Networks

Jochen Renz¹ and Stefan Wölfl²

Abstract. Route navigation is one of the most widely used everyday application of spatial data. In this paper we investigate how a qualitative representation of route networks can be derived from map data and how this representation can be used to reason about route descriptions. We introduce a concept of route graph that provides an abstract layer on top of metric map data and thus allows for a compact representation of route information. We present selected queries and reasoning tasks that can be expressed in this abstraction layer.

1 Introduction

Route navigation is one of the most central everyday application of spatial data. With the broad availability of GPS receivers, people can record their own routes and upload them to navigation services. There is a wide range of possible uses for these GPS data: they allow for improving classical navigation services for street networks, but also for generating user-centric route maps that take into account individual preferences or maps that are tailored to specific activities (e.g., mountain biking). One area that seems particular useful is the generation of route descriptions that are easy to understand, be remembered, and processed by users. Since qualitative representations promise to offer these features, it seems natural to have a qualitative representation of routes and route networks.

There are several approaches on how graph-like structures can be used to reason about spatial data. For example, in [1] so-called travel graphs (labeled weighted graph representing streets, crossings, and distance information) are introduced to deal with the route planning problem and the problem of inferring a map layout for a street network from noisy GPS data. More closely related to our work is the concept of route graph presented in [3], which can be used for representing route instructions in the context of human-robot interaction. Cognitive results on qualitative route instructions are reported in [2].

In this paper we will sketch some first ideas how qualitative representation and reasoning techniques can be applied to solve tasks related to route navigation and route descriptions. Thereto, we introduce a concept of route graph that provides a qualitative, abstract layer on top of metric map data.

2 From Maps to Route Graphs

Geodata as for example stored on OpenStreetMap define a map-like graph structure that represents parts of the street network. That is, they specify nodes associated to some point in the network and paths between nodes associated to some navigable way between points in the network. From these data, one can extract (directed) arcs (v_i, v_j) between nodes in the street network. We assume that distinct arcs

 $(v_i, v_j), (v'_i, v_j)$ represent distinct possibilities to enter v_j in the represented street network (and accordingly for different arcs starting in the same node). Then, an intersection node can be defined as a node that is incident to at least three other nodes, i.e., these nodes occur in arcs that start or end at the intersection node. All other nodes may be referred to as connection nodes (or: contour nodes). A path arc is a sequence (x, \vec{c}, y) of nodes defining a directed path, where x and y are intersection nodes and $\vec{c} = (c_1, \ldots, c_n)$ is a possibly empty sequence of contour nodes. In the format of OpenStreetMap, for example, such path arcs (x, c_1, \ldots, c_n, y) occur as sub-sequences of so-called way specifications. To cast this into a concept, one can define: Given a set of map data D, a map graph on D is a tuple $G_D = \langle I, C, PA, \rho \rangle$, where I is the set of intersection nodes in D, C is the set of its contour nodes, PA is the set of its path arcs (i.e., each path arc is a an element of $I \times C^n \times I$ for some $n \ge 0$), and $\rho: PA \to 2^{PA}$ is the function that assigns to each path arc (x, \vec{c}, y) the (possibly empty) set of path arcs (y, \vec{c}', z) in which the intersection y can be left, when entered from (x, \vec{c}, y) .

Map graphs offer a representation that is very close to the actual geodata. However, when representing a route and describing it to a user, the most important information is when and where to turn when driving along the route. A route instruction at an intersection could include action terms that refer to qualitative direction relations, such as "turn sharp left", "turn right", "turn around", "go straight". Our goal here is to define a qualitative representation of route networks that abstracts from the concrete geodata as used in map graphs and directly builds on such qualitative relations. This allows for a straight-forward derivation of route descriptions and for reasoning about route instructions on a symbolic level. Hence, the idea is to generate a graph structure that only contains the intersection nodes used in map graphs, but employs the geometric information about contour nodes (in fact, the first and last contour node in a path arc) to determine the qualitative spatial relations that can be used to express spatial actions at an intersection.

Qualitative relations occurring in such route descriptions can be defined in terms of spatial relations as discussed in the field of Qualitative Spatial Reasoning [6]. Here we are particularly interested in spatial relations defined on points in the Euclidean plane (given some standard projection of the geodata into the plane). Let \mathcal{R} be a set of jointly exhaustive and pairwise disjoint direction relations that describe the position of a point z relative to a directed line (x, y). Then, a *route graph* $G_{\mathcal{R}}$ over \mathcal{R} is a tuple $\langle V, A, r \rangle$, where V is a nonempty, finite set of nodes, A is a set of arcs on V, and r is a function that assigns a spatial relation from \mathcal{R} to each pair of arcs $(x, y), (y, z) \in A$.

Route graphs can be defined from map graphs in a straight-forward way. Moreover, there are several reasoning problems that are related to route graphs, for example: Given a route graph $G_{\mathcal{R}}$, is there a map graph whose corresponding route graph is $G_{\mathcal{R}}$? This problem can be simply cast as a constraint satisfaction problem with variables for intersection nodes, auxiliary variables for contour nodes, and constraints with relations from \mathcal{R} .

¹ School of Computer Science, The Australian National University, Australia.

² Department of Computer Science, University of Freiburg, Germany. We acknowledge support by DFG (*SFB/TR 8 Spatial Cognition*, project R4-[LogoSpace]), an ARC Future Fellowship (FT0990811), and a joint Go8/DAAD project (D/08/13855).

3 A Qualitative Representation of Routes

The most common qualitative direction relations over points in the plane depend on the angles formed by the points, where angles that yield the same direction relation belong to a common direction sector bounded by different angles. There exist two families of qualitative formalisms (calculi) that allow for defining sectors with different angles: the STAR calculi [5] for absolute direction relations and the OPRA calculi [4] for relative direction relations. For a given STAR calculus, the direction sectors are the same for every point p on the plane, while the sectors of a given OPRA calculus depend on the orientation of p. These sectors plus p itself (which forms the 0 sector) are the *base relations* of the STAR or OPRA calculus S, we assume the sectors are numbered in clockwise direction from 1 to m. Any two points q, r that are in the same sector i with respect to p are in the same qualitative direction relation with p. The number of sectors m and their angles can be arbitrarily chosen based on the requirements of the application and can then be applied to all points in the plane. For the purpose of this paper, we assume that all sectors are point symmetric and each covers a range of 360/m degrees (see Figure 1(a)). Even though we deal with relative direction relations, in this paper we use STAR calculi for representing direction sectors in order to have a consistent sector arrangement for every intersection node. OPRA would require different sector arrangements for the same intersection node whenever there are multiple incoming paths.

For a given STAR calculus S we can easily compute the STAR relation of a path arc $a = (x, c_1, \ldots, c_n, y) \in A$ with respect to either intersection node x or y as the sector of x that contains c_1 or the sector of y that contains c_n , respectively. We write the STAR relation of a wrt a node v as s(a, v). If v is not part of a, then s(v, a)is undefined. Since a STAR calculus allows the definition of different sector arrangements, one task is to determine the STAR calculus that is best suited for a given map graph G_D . This can be formulated as the following reasoning task: Given a map graph, determine the coarsest STAR calculus S that allows for distinguishing different outgoing edges in G_D in a unique way.

In order to use S for qualitative turn descriptions of a route, we have to define a mapping $t: V \times V \times V \mapsto \mathcal{L}$ that maps an incoming arc $a_i = (x, v)$ and an outgoing arc $a_o = (v, y)$ at an intersection node v (in fact, the two corresponding STAR relations $s(a_i, v)$ and $s(a_o, v)$) to an element of a set of *qualitative turn labels* \mathcal{L} . When an arc enters an intersection v from a sector $s(a_i, v)$ then each sector is assigned a qualitative turn label according to which turn it represents with respect to $s(a_i, v)$, for example, "left", "right", "slightly left", "sharp left", "back", "straight", and "stop". Different sectors can be mapped to the same label (see Figure 1(b) for an example). Since this mapping is rotation invariant (see Figure 1(c)), we can define a function $f(a_i, v, a_o) = s(a_o, v) - s(a_i, v) \mod m$ that considers s as the numerical value of the sectors. If $f(a_i, v, a_o) = i$ then a_o is *i* sectors in clockwise direction from $s(a_i, v)$. The mapping t only depends on the value of f except for when $s(a_o, v) = 0$ in which case t(x, v, y) ="stop". For instance, if $f(a_i, v, a_o) = m/2$, then t(x, v, y) should be "straight". The set of qualitative turn labels \mathcal{L} and the mapping t can now be used to define a route graph $G_{\mathcal{R}}$ = (V, E, r) where $\mathcal{R} = \mathcal{L}$ and r = t.

A qualitative route description T of a route $R = (p_1, \ldots, p_n)$ is then a sequence of turn labels $T = (t(p_1, p_2, p_3), t(p_2, p_3, p_4), \ldots, t(p_{n-2}, p_{n-1}, p_n))$. In situations where we are only interested in turns, we can restrict a qualitative route description to those intersection points where an actual turn occurs. We call such a reduced qualitative route description a qualitative turn description. By counting how many "straight" labels occur between turns, we can also extract descriptions such as "turn left at the 5th intersection".



Figure 1. (a) STAR relations, (b) qualitative turn labels assigned to STAR relations, (c) an incoming arc and three possible outgoing arcs.

4 Reasoning about Qualitative Route Descriptions

Related to route descriptions, there are different reasoning tasks or queries of interest, for example, given a route graph $G_{\mathcal{R}}$:

- Given a qualitative route/turn description, find one/all routes in $G_{\mathcal{R}}$ that match this description.
- Given a qualitative route/turn description and a starting point, what are the possible end points of the route (and vice versa)?
- Given a start and end point, what is the route with the least turns?

The most basic query, however, is the (decision) problem whether a route description $d = (d_1, \ldots, d_n)$ can be instantiated in a route graph (given a fixed start arc (v_0, v_1) as well as a goal node v_q). This problem can be tackled, for example, by using a queue that stores pairs of arcs and trailing sequences of d. Initially this queue just contains $((v_0, v_1), d)$. As long as this queue is not empty, we select and delete one element from the queue $((v_i, v_j), (d_l, \ldots, d_n))$, and try to execute d_l at v_i . If that is possible, we add those pairs $((v_i, v_k), (d_{l+1}, \ldots, d_n))$ to the queue where (v_i, v_k) is in fact an arc in the graph. Otherwise, we add pairs $((v_i, v_k), (d_1, \ldots, d_n))$ to the queue and do the same for each node that is in the "straight" direction w.r.t. the arc (v_i, v_j) . If we reach the goal node with an empty route description, we can stop and return "yes". Otherwise, if the queue becomes empty without reaching the goal node, we return "false". If one ensures that an arc-description pair is never processed twice (e.g., by marking processed pairs), this algorithm is guaranteed to terminate and runs in time $\mathcal{O}(e \cdot n)$ where e is the number of arcs in the graph and n is the length of the route description.

5 Future Work

We have mentioned a number of interesting reasoning tasks and queries that can be analyzed within our framework. In addition, our approach forms the basis for other qualitative representations, such as relations between routes and route segments. We expect that this leads to more intuitive route descriptions and eventually to improved and more user-friendly navigation systems.

REFERENCES

- S. Edelkamp and S. Schrödl, 'Route planning and map inference with global positioning traces', in *Computer Science in Perspective*, LNCS 2598, pp. 128–151. (2003).
- [2] A. Klippel, C. Dewey, M. Knauff, K.-F. Richter, D. R. Montello, C. Freksa, and E.-A. Loeliger, 'Direction concepts in wayfinding assistance systems', in *Proc. AIMS'04*, SFB 378 Memo 84, pp. 1–8, (2004).
- [3] B. Krieg-Brückner and H. Shi, 'Orientation calculi and route graphs: Towards semantic representations for route descriptions', in *GIScience'06*, pp. 234–250. (2006).
- [4] R. Moratz, 'Qualitative spatial reasoning about oriented points', Technical Report 003/10-2004, SFB/TR 8 Spatial Cognition, (2004).
- [5] J. Renz and D. Mitra, 'Qualitative direction calculi with arbitrary granularity', in *PRICAI'04*, pp. 65–74. (2004).
- [6] J. Renz and B. Nebel, 'Qualitative spatial reasoning using constraint calculi', in *Handbook of Spatial Logics*, 161–215, Springer. (2007).