Hierarchical Modulation-based Cooperative Scheme: Minimizing the Symbol Error Probability

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***Abstract*— In a cooperative network, relays are often closer to the source than the destination and thus the source-relay links have a higher capacity than the source-destination one. In this paper, we investigate a hierarchical modulation-based cooperative scheme, where the relay is able to detect the whole signal, while the destination can only detect the first layer of the modulation. We provide an error probability upper-bound for this scheme, which allows us to optimize the choice of the constellation parameter θ.**

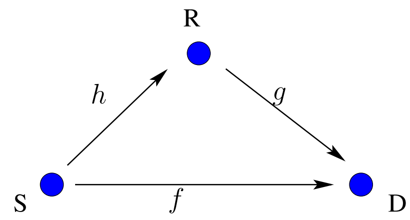

Fig. 1. Relay channel model.

I. Introduction

In the last decade there has been a growing interest in wireless cooperative communications [1], [2], [3]. Indeed wireless nodes cannot always be equipped with several antennas due to size, cost or hardware limitations. But some diversity can still be exploited by considering the virtual multiple antenna array formed by several nodes of the network and using distributed space-time transmission techniques.

In such cooperative networks, the relays are often closer to the source than the destination. Thus they should be able to detect a higher amount of information. Hierarchical modulation (HM) was originally used as an unequal error protection technique which has been extensively used in DVB-T [4], [5]. In [6], [7], this unequal error protection technique has been generalized to the relay channel. The idea of sending additive information to the relay using the second layer of a HM has been proposed in [8]. However the authors considered a broadcast channel with two non-cooperative receivers, and thus did not exploit the diversity of the wireless channel. A HM-based cooperative protocol has been proposed in [9] and further studied in [10]. The authors proposed to exploit diversity by implementing a distributed Alamouti code [11] and studied the simulation performance of this protocol. They showed by simulation that performance is impacted by the choice of the HM parameter.

In this paper, we propose a modification of this last protocol that facilitates the theoretical study of the performance. We provide an upper-bound on the symbol error probability of this cooperative scheme which is shown by simulation to be within 1 dB of the real optimal performance. This upper-bound allows us to find the optimal parameter θ^* of the HM that optimizes the performance.

A. Channel model

We consider a cooperative network with one source, one relay and one destination. All nodes are full-duplex and equipped with a single antenna. We assume that the capacity of the source-relay link is higher than the source-destination one.

In this paper, notations of Fig. 1 are used. The fading channel links between source and destination, relay and destination, and source and relay are denoted by f, g and h, and follow Rayleigh distributions with variances σ_f^2, σ_g^2 and σ_h^2, respectively. The channel is assumed to be slow-fading. Thus f, g and h can be considered as constant during the transmission of at least one codeword.

The power is uniformly distributed between the source and the destination as our aim in this paper is not optimizing power allocation.

B. Hierarchical modulation

Since the quality of the source-relay link is on average better, the relay is able to detect a higher amount of information than the destination. Thus a HM can be used by the source, where the first layer of the modulation is for the destination, but the second layer is detected only by the relay.

This HM is represented in Fig. 2. The big blue points $x_i, i \in \{1, \dots, 4\}$ belong to the QPSK constellation which is detected by the destination. The small red points $s_k, k \in \{1, \dots, 16\}$ belong to the HM constellation and are detected by the relay only. The position of these small red points depends on the HM parameter θ. They can be expressed as the parameterized sum of two points of the QPSK constellation. For example, $s_5 = \frac{x_2+\theta x_1}{\sqrt{1+\theta^2}}$, where the factor $\frac{1}{\sqrt{1+\theta^2}}$ comes from the normalization of the signal power.

In particular, one can notice that the center of the 4 red points in the same quadrant is not the QPSK point. A special value of θ is 0.5, for which we obtain the usual 16-QAM constellation.

More details on HM can be found in [4].

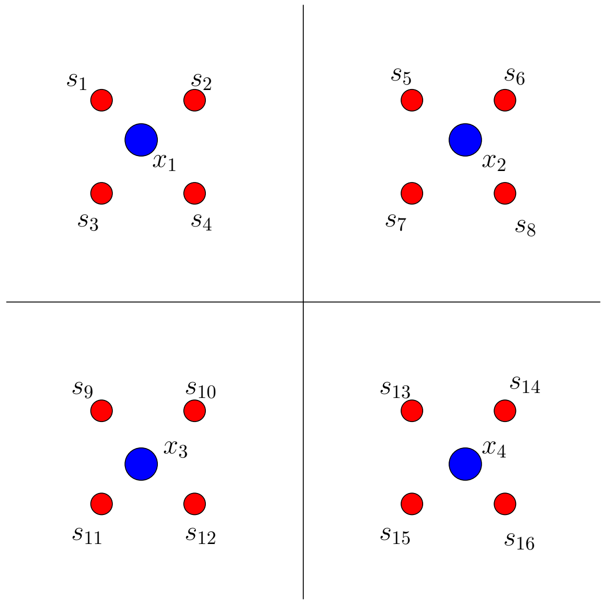

Fig. 2. Hierarchical modulation.

II. HM-BASED COOPERATIVE SCHEME

The HM-based cooperative protocol studied in this paper is similar to the one proposed in [9] and further studied in [10]. However, a slight modification of the signals sent by the relay has been introduced in order to simplify the theoretical analysis. In Section V, simulations will show that this modification has a very low impact on the symbol error probability of the protocol.

In order to exploit the diversity of the wireless channel, the simplest space-time block code, i.e. the Alamouti code [11], is used. The transmission frame is described in Tab. I:

- the source transmits the first QPSK symbol to the relay in order to initiate the transmission;
- Alamouti codewords are then sent in a distributed manner: however, instead of sending x_{2i} at the beginning of each codeword, the source sends the HM symbol $t_{2i,2i+1} = \frac{t_{2i}+\theta t_{2i+1}}{\sqrt{1+\theta^2}}$. The destination still sees this as the QPSK symbol t_{2i}, but the relay is able to detect both t_{2i} which is necessary for the current codeword and t_{2i+1} which is necessary for the next one. In the protocol proposed in [9], the relay retransmits the detected QPSK symbol $\tilde{t}_{2i}$, however, in this paper, we choose to retransmit the detected HM symbol $\tilde{t}_{2i,2i+1}$, in order to keep the Alamouti code structure and facilitate the theoretical analysis;
- since no more symbols have to be sent in advance to the relay, the last codeword is a usual one, containing only QPSK symbols.

At destination, the message can be decoded using the usual Alamouti decoding method. Received signals in time slots

TABLE I

TRANSMISSION FRAME: BLUE AND RED SENT SYMBOLS CORRESPOND TO BLUE AND RED POINTS OF THE HIERARCHICAL MODULATION IN FIG. 2

time	source	relay
0	t_1	
1	$t_{2,3}$	$\tilde{t}_1$
2	$-t_1^*$	$\tilde{t}_{2,3}^*$
...	...	...
$2i-1$	$t_{2i,2i+1}$	$\tilde{t}_{2i-1}$
$2i$	$-t_{2i-1}^*$	$\tilde{t}_{2i,2i+1}^*$
...	...	...
$N-3$	$t_{2N-2,2N-1}$	$\tilde{t}_{2N-3}$
$N-2$	$-t_{2N-3}^*$	$\tilde{t}_{2N-2,2N-1}^*$
$N-1$	t_{2N}	$\tilde{t}_{2N-1}$
N	$-t_{2N-1}^*$	$\tilde{t}_{2N}^*$

$2i-1$ and $2i$ are:

$$y_{2i-1} = \sqrt{E_s}(ft_{2i,2i+1} + g\tilde{t}_{2i-1}) + w_{2i-1},$$
$$y_{2i} = \sqrt{E_s}(-ft_{2i-1}^* + g\tilde{t}_{2i,2i+1}^*) + w_{2i}.$$

where E_s is the signal power and w is additive white Gaussian noise (AWGN) with variance σ_w^2.

Let us assume that the signals have been correctly detected at relay, then, we can rewrite the previous equations as a multiple antenna system:

$$\begin{bmatrix} y_{2i-1} \\ y_{2i}^* \end{bmatrix} = \sqrt{E_s} \underbrace{\begin{bmatrix} g & f \\ -f^* & g^* \end{bmatrix}}_{\mathbf{H}} \begin{bmatrix} t_{2i-1} \\ t_{2i,2i+1} \end{bmatrix} + \begin{bmatrix} w_{2i-1} \\ w_{2i}^* \end{bmatrix}$$

$$\frac{1}{\sqrt{|f|^2+|g|^2}}\mathbf{H}^\dagger \begin{bmatrix} y_{2i-1} \\ y_{2i}^* \end{bmatrix} = \sqrt{E_s}\sqrt{|f|^2+|g|^2} \begin{bmatrix} t_{2i-1} \\ t_{2i,2i+1} \end{bmatrix} + \underbrace{\frac{1}{\sqrt{|f|^2+|g|^2}}\mathbf{H}^\dagger \begin{bmatrix} w_{2i-1} \\ w_{2i}^* \end{bmatrix}}_{\mathbf{w}} \quad (1)$$

where for fixed f and g, $\mathbf{w}$ is AWGN with the same variance σ_w^2 per element.

III. AN UPPER-BOUND ON THE ERROR PROBABILITY OF THE HM-BASED COOPERATIVE SCHEME

In order to consider optimal performance, we assume that the destination knows if the relay is able to decode. If it is not, the relay remains silent and the destination detects the message based on source signals only. The error probability of the protocol is then given by:

$$p_e = (1-\alpha p_e^r)p_e^{coop} + \alpha p_e^r p_e^{siso} \quad (2)$$

where p_e^r is the error probability at relay, p_e^{coop} is the error probability at destination when the cooperative scheme is used and p_e^{siso} is the error probability at destination when the relay is not transmitting. The factor $\alpha \in [1,2]$ depends on p_e^r. Indeed, when an HM symbol is incorrectly decoded at relay, cooperation cannot be used not only for the current codeword, but also for the next one. When p_e^r tends to 0, there is a greater chance that the errors are isolated and thus α tends to 2. On

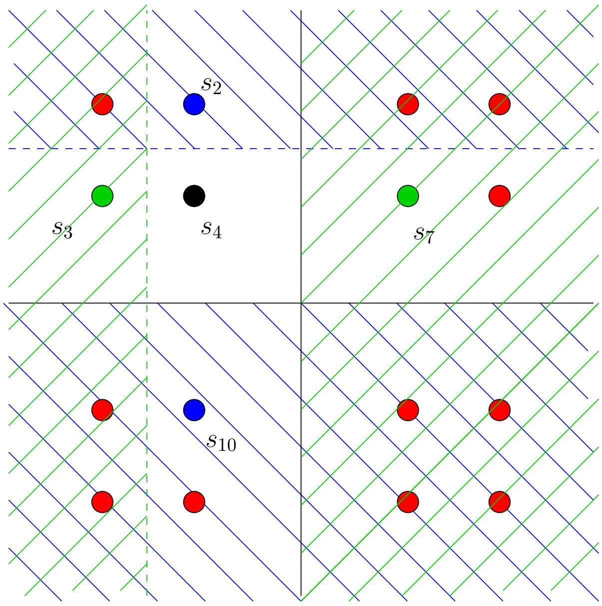

Fig. 3. Error region if the sent signal is s_4. If the received signal belongs to the green region, it is closer to s_3 or s_7 than s_4 and a decoding error occurs. If it belongs to the blue region, it is closer to s_2 or s_{10}.

the contrary, if p_e^r tends to 1, there is a greater chance that the errors are clustered and thus α tends to 1.

A straight upper-bound on the error probability is then:

$$p_e \leq p_e^{coop} + 2p_e^r p_e^{siso} \tag{3}$$

A. Error probability at relay

Signals listened to by the relay belong to the HM constellation (small red points on Fig. 2). The relay, which is close to the source, is hopefully able to detect these signals.

The HM constellation contains 16 symbols. Thus, for a given sent signal, there are 15 decoding error events, each of them corresponding to an incorrect point of the HM constellation. Assuming equiprobable messages, the probability of an error occurring at relay is given by the probability of the union of these 15 error events.

$$p_e^r = \frac{1}{16}\sum_{i=1}^{16} \Pr\left(\bigcup_{j\neq i}\{s_i \to s_j\}\right) \tag{4}$$

As can be seen on Fig. 3, by considering the pairwise error events with only the closest points to the sent signal, the whole error region is covered, thus the error probability at relay can be rewritten:

$$p_e^r = \frac{1}{16}\sum_{i=1}^{16} \Pr\left(\bigcup_{s_j \text{neighbor of} s_i}\{s_i \to s_j\}\right) \tag{5}$$

Using the union bound, we obtain:

$$p_e^r \leq \frac{1}{16}\sum_{i=1}^{16} \sum_{s_j \text{neighbor of} s_i} \Pr\left(s_i \to s_j\right). \tag{6}$$

For a given channel realization, the probability of detecting s_j if the symbol s_i has been sent is given by:

$$\begin{aligned}
&\Pr(s_i \to s_j|h) = \Pr(|y - \sqrt{E_s}hs_i|^2 \geq |y - \sqrt{E_s}hs_j|^2) \\
&= \Pr(|w|^2 \geq |\sqrt{E_s}h(s_i - s_j) + w|^2) \\
&= \Pr(\underbrace{|\sqrt{E_s}h(s_i - s_j)|^2 + 2\Re(\sqrt{E_s}h(s_i - s_j)w^*)}_{V} \leq 0).
\end{aligned}$$

V is a Gaussian random variable with mean $m_V = E_s|h(s_i - s_j)|^2$ and variance $\sigma_V^2 = 4E_s|h(s_i - s_j)|^2\sigma_w^2$. The pairwise error probability is thus given by:

$$\Pr(s_i \to s_j|h) = Q\left(\frac{m_V}{\sigma_V}\right) = Q\left(\frac{|\sqrt{E_s}h(s_i - s_j)|}{2\sigma_w}\right). \tag{7}$$

Finally, averaging over all channel realizations, we obtain:

$$\Pr(s_i \to s_j) = E_h\left\{Q\left(\frac{|\sqrt{E_s}h(s_i - s_j)|}{2\sigma_w}\right)\right\}. \tag{8}$$

In this case, integration is possible and we obtain an exact expression for the pairwise error probability:

$$\Pr(s_i \to s_j) = \frac{1}{2}\left(1 - \sqrt{\frac{|s_i - s_j|^2\rho\sigma_h^2/4}{1 + |s_i - s_j|^2\rho\sigma_h^2/4}}\right), \tag{9}$$

where $\rho = \frac{E_s}{\sigma_W^2}$.

Since this expression depends only on the distances between the sent and detected symbols, we can simplify the expression of the error probability at relay:

$$p_e^r \leq 2\Pr(s_2 \to s_1) + \Pr(s_2 \to s_5), \tag{10}$$

with $|s_2 - s_1|^2 = \frac{2\theta^2}{1+\theta^2}$ and $|s_2 - s_5|^2 = \frac{2(1-\theta)^2}{1+\theta^2}$.

B. Error probability at destination when only the direct link between source and destination is used

When the relay is not able to detect the source signals, a cooperative strategy cannot be used, since the relay would forward errors. Thus the relay remains silent and the destination receives signals only from the source.

Half of these signals (even time slots) belong to a QPSK. The other half (odd time slots) belong to the HM constellation. The total error probability is thus the mean of these two cases:

$$p_e^{siso} = \frac{1}{2}\left(p_{e,even}^{siso} + p_{e,odd}^{siso}\right) \tag{11}$$

1) The sent signal belongs to the QPSK: In this case, an upper-bound to the error probability is easy to compute. Considering only the pairwise error probabilities for neighbors and noting that some distances are equal:

$$p_{e,even}^{siso} \leq 2\Pr(x_1 \to x_2), \tag{12}$$

where similarly to (9) and noting that $|x_1 - x_2|^2 = 2$, the pairwise error probability is given by

$$\Pr(x_1 \to x_2) = \frac{1}{2}\left(1 - \sqrt{\frac{\rho\sigma_f^2/2}{1 + \rho\sigma_f^2/2}}\right). \tag{13}$$

2) The sent signal belongs to the HM constellation: The destination, however, still wants to detect the corresponding QPSK symbol. The expression of the error has then to be redefined. Let us assume that the signal s_k is actually sent. The destination wants to detect x_i, $i = \lceil \frac{k}{4} \rceil$, where $\lceil . \rceil$ denotes the integer ceiling function.

$$\begin{aligned}
&\Pr(x_i \to x_j | f, s_k) = \Pr(|y - \sqrt{E_s} f x_i|^2 \geq |y - \sqrt{E_s} f x_j|^2) \\
&= \Pr(|\sqrt{E_s} f(s_k - x_j) + w|^2 - |\sqrt{E_s} f(s_k - x_i) + w|^2 \leq 0) \\
&= \Pr(|\sqrt{E_s} f(s_k - x_j)|^2 + 2\Re(\sqrt{E_s} f(s_k - x_j) w^*) \\
&\qquad - |\sqrt{E_s} f(s_k - x_i)|^2 - 2\Re(\sqrt{E_s} f(s_k - x_i) w^*) \leq 0) \\
&= \Pr(E_s |f|^2 (|s_k - x_j|^2 - |s_k - x_i|^2) \\
&\qquad - 2\Re(\sqrt{E_s} f(x_j - x_i) w^*) \leq 0)
\end{aligned}$$

V, the left term of the inequality, is a Gaussian random variable with mean $m_V = E_s |f|^2 (|s_k - x_j|^2 - |s_k - x_i|^2)$ and variance $\sigma_V^2 = 4E_s |f(x_j - x_i)|^2 \sigma_W^2$. The conditioned pairwise error probability can thus be written:

$$\Pr(x_i \to x_j | f, s_k) = Q\left(\frac{\sqrt{E_s}|f|}{2\sigma_W} \frac{|s_k - x_j|^2 - |s_k - x_i|^2}{|x_j - x_i|}\right) \tag{14}$$

Averaging over all channel realizations, we obtain

$$\Pr(x_i \to x_j | s_k) = E_f \left\{ Q\left(\frac{\sqrt{E_s}|f|}{2\sigma_W} \frac{|s_k - x_j|^2 - |s_k - x_i|^2}{|x_j - x_i|}\right)\right\} \tag{15}$$

whose exact expression is given by (9) where we replace the distance $|s_i - s_j|$ by $\frac{|s_k - x_j|^2 - |s_k - x_i|^2}{|x_j - x_i|}$ and the variance of h by that of f. To obtain the pairwise error probability, we average this expression over the sent signals:

$$\Pr(x_i \to x_j) = \frac{1}{4} \sum_{k=4(i-1)+1}^{4(i-1)+4} \Pr(x_i \to x_j | s_k). \tag{16}$$

And finally, we obtain a simplified upper-bound on the error probability by considering the closest points to the sent signal and noting that some distances are the same:

$$p_{e,odd}^{siso} \leq \frac{1}{2} \sum_{k=1}^{4} \Pr(x_1 \to x_2 | s_k), \tag{17}$$

with $\frac{(|s_1 - x_2|^2 - |s_1 - x_1|^2)^2}{|x_2 - x_1|^2} = \frac{(|s_4 - x_2|^2 - |s_4 - x_1|^2)^2}{|x_2 - x_1|^2} = \frac{2(1-\theta)^2}{1+\theta^2}$ and $\frac{(|s_2 - x_2|^2 - |s_2 - x_1|^2)^2}{|x_2 - x_1|^2} = \frac{(|s_3 - x_2|^2 - |s_3 - x_1|^2)^2}{|x_2 - x_1|^2} = \frac{2(1+\theta)^2}{1+\theta^2}$.

C. Error probability at destination when cooperation is used

When the relay is able to detect the source signals, a cooperative strategy is used. Thanks to the use of the Alamouti code, the MIMO decoding is reduced the two SISO decodings, one of them in a QPSK, the other one in the HM constellation. In this case again, the error probability is the mean of both cases:

$$p_e^{coop} = \frac{1}{2}\left(p_{e,even}^{coop} + p_{e,odd}^{coop}\right). \tag{18}$$

However, the channel is not a Rayleigh-distributed coefficient any more, but of the form $\sqrt{|f|^2 + |g|^2}$.

1) The sent signal belongs to the QPSK: When the sent signal is a QPSK symbol, the pairwise error probability is given by:

$$\Pr(x_i \to x_j) = E_{f,g}\left\{ Q\left(\frac{\sqrt{|f|^2 + |g|^2}|x_i - x_j|}{2\sigma_W}\right)\right\}. \tag{19}$$

We can obtain the exact expression of this probability by integrating over f and g. The result depends on the variances of these two channels. We can distinguish two cases, whether they are equal or not (see appendix A for the derivation of these expressions).

If $\sigma_f \neq \sigma_g$, the pairwise error probability is given by:

$$\begin{aligned}
\Pr(x_i \to x_j) = \frac{1}{2}\Bigg(1 &- \frac{\sigma_f^2}{\sigma_f^2 - \sigma_g^2}\sqrt{\frac{|x_i - x_j|^2 \rho \sigma_f^2/4}{1 + |x_i - x_j|^2 \rho \sigma_f^2/4}} \\
&- \frac{\sigma_g^2}{\sigma_g^2 - \sigma_f^2}\sqrt{\frac{|x_i - x_j|^2 \rho \sigma_g^2/4}{1 + |x_i - x_j|^2 \rho \sigma_g^2/4}}\Bigg).
\end{aligned} \tag{20}$$

If $\sigma_f = \sigma_g$, the pairwise error probability is given by:

$$\begin{aligned}
\Pr(x_i \to x_j) = \frac{1}{2}\Bigg(1 &- \sqrt{\frac{|x_i - x_j|^2 \rho \sigma_f^2/4}{1 + |x_i - x_j|^2 \rho \sigma_f^2/4}} \\
&- \frac{1}{2}\frac{\sqrt{|x_i - x_j|^2 \rho \sigma_f^2/4}}{(1 + |x_i - x_j|^2 \rho \sigma_f^2/4)^{\frac{3}{2}}}\Bigg).
\end{aligned} \tag{21}$$

An upper-bound on the error probability is then:

$$p_{e,even}^{coop} \leq 2\Pr(x_1 \to x_2). \tag{22}$$

2) The sent signal belongs to the HM constellation: In this case, we obtain expressions for the pairwise error probabilities that are very similar to (20) and (21), except that the distance $|x_i - x_j|$ has to be replaced by $\frac{|s_k - x_j|^2 - |s_k - x_i|^2}{|x_j - x_i|}$:

$$p_{e,odd}^{coop} \leq 2\Pr(x_1 \to x_2) = \frac{1}{2}\sum_{k=1}^{4} \Pr(x_1 \to x_2 | s_k). \tag{23}$$

IV. Choice of θ

On one hand, if θ is large, it will be easier for the relay to decode, but there will be more decoding errors at destination. On the other hand, if θ is small, destination will be able to easily detect QPSK symbols, but it will be more difficult for the relay to decode. Thus a tradeoff has to be found that will minimize the overall error probability (3).

Fig. 4 represents the performance of the cooperative scheme in terms of symbol error rate (SER) as a function of θ for several signal-to-noise ratios (SNR). It shows that the optimal choice of θ depends on the SNR. However, this dependence is not significant and the performance for values of θ close to the optimal are very similar. Thus it makes sense to get rid of this dependence by choosing the optimal θ for the high SNR regime.

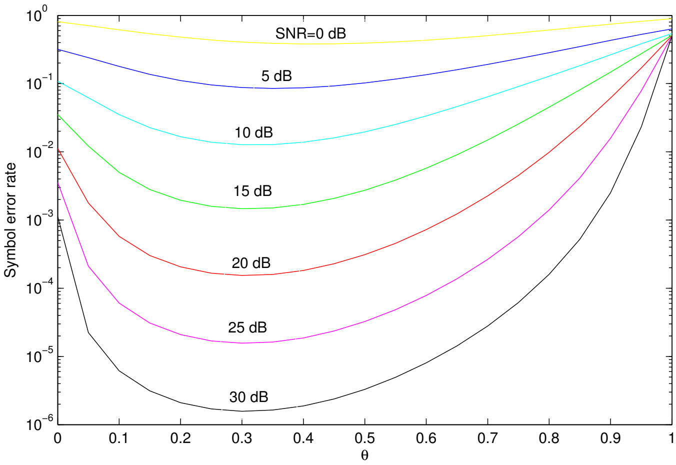

Fig. 4. Symbol error rate of the HM-based protocol as a function of the constellation parameter θ. The optimal choice of θ slightly depends on the SNR.

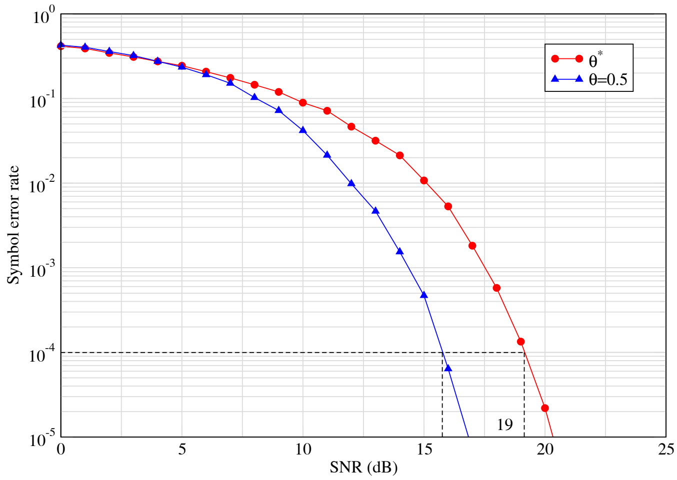

Fig. 5. Symbol error rate of a Gaussian link when sent signals belong to an HM constellation.

Noting that $\sqrt{\frac{x}{1+x}} = 1 - \frac{1}{2x} + \frac{3}{8x^2} + \mathcal{O}\left(\frac{1}{x^3}\right)$ when $x \to \infty$, we obtain the asymptotic behavior of the error probability (9) when the SNR is high:

$$\Pr(s_i \to s_j) \doteq \frac{1}{|s_i - s_j|^2 \rho \sigma_h^2}. \tag{24}$$

The same approximation can be used at destination in the non-cooperative case. We only have to change the distance by the appropriate one ($|x_i - x_j|$ if a QPSK symbol is considered, $\frac{|s_k - x_j|^2 - |s_k - x_i|^2}{|x_j - x_i|}$ otherwise) and the variance by the source-destination link one σ_f^2.

In the same way, and also noting that $\frac{1}{1+x} = \frac{1}{x} - \frac{1}{x^2} + \mathcal{O}\left(\frac{1}{x^3}\right)$ when $x \to \infty$, we obtain the asymptotic behavior of the error probabilities (20) and (21) when the SNR is high. If $\sigma_f^2 \neq \sigma_g^2$,

$$\Pr(x_i \to x_j) \doteq 3 \frac{\sigma_f^2 + \sigma_g^2}{\sigma_f^4 \sigma_g^4} \frac{1}{\rho^2 |x_i - x_j|^4} \tag{25}$$

and if $\sigma_f^2 = \sigma_g^2$,

$$\Pr(x_i \to x_j) \doteq 3 \frac{1}{\rho^2 |x_i - x_j|^4 \sigma_f^4}. \tag{26}$$

If an HM symbol has been sent, the same approximations can be used, replacing $|x_i - x_j|$ by $\frac{|s_k - x_j|^2 - |s_k - x_i|^2}{|x_j - x_i|}$.

The minimization problem can be translated into a high degree polynomial equation. Using a solver such as Matlab or Mathematica, the optimal θ^* can be found. Some values of θ^* are given in Tab. II for different channels.

We can notice that the optimal value of θ increases with the capacity of the relay-destination link and decreases with the capacity of the source-relay link, which agrees with intuition.

TABLE II
OPTIMAL VALUES OF θ

σ_f^2 (dB)	σ_g^2 (dB)	σ_h^2 (dB)	θ^*
0	0	0	0.387
0	0	5	0.350
0	0	10	0.301
0	5	5	0.380
0	5	10	0.339
0	10	10	0.384

V. PERFORMANCE

We assume that the relay is close to the source and that the source-destination and relay-destination distances are equal. We translate this network topology by a 10 dB gain on the source-relay link, while the source-destination and relay-destination variances are $\sigma_f^2 = \sigma_g^2 = 0$ dB. Referring to Tab. II, the optimal value of θ in this case is $\theta^* = 0.301$. In the following, simulations are run for this optimal θ^*.

Fig. 6 represents the SER performance of the protocol as a function of the SNR. Two cases have been considered for the simulation:

- the unrealistic case where the genie-aided relay forwards only if it has correctly decoded the signal. This case is not implementable in practice but gives the optimal performance. It shows that the theoretical upper-bound is within 1 dB, except for SNR lower than 5 dB. Indeed, in this case, the error probability at relay is not negligible and the upper-bound $1 - \alpha p_e^r \leq 1$ is loose.
- the practical case where the relay chooses to forward according to an SNR criterion. A target probability of error is chosen at relay, for example $p_e^r = 10^{-4}$. For θ^*, the corresponding SNR for a Gaussian link (19 dB) can be read on Fig. 5. The relay retransmits the information if the received SNR at relay is greater than 19 dB:

$$(\rho |h|^2)_{dB} > 19. \tag{27}$$

This SNR criteria induces a loss in performance of around 3 dB.

Fig. 7 shows that our modification of the protocol proposed in [9] does not have a big impact on the performance. Indeed, it induces only a 1 dB loss for the optimal performance, and

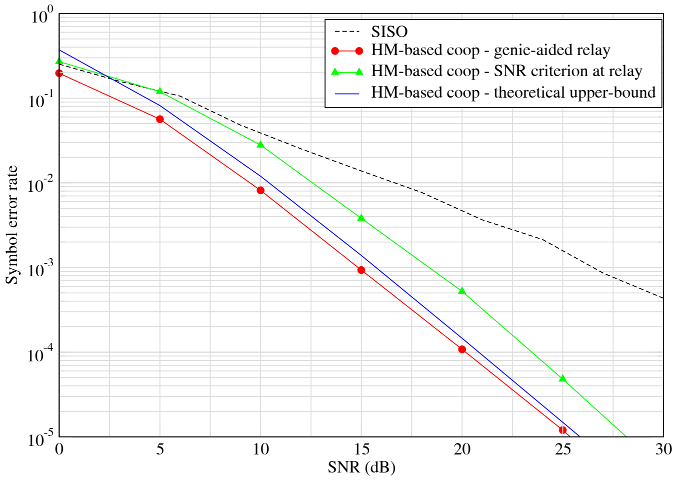

Fig. 6. Theoretical and simulation-based symbol error rates of the HM-based cooperative scheme as a function of the SNR. The proposed upperbound on the symbol error rate probability is within 1 dB of the real optimal performance.

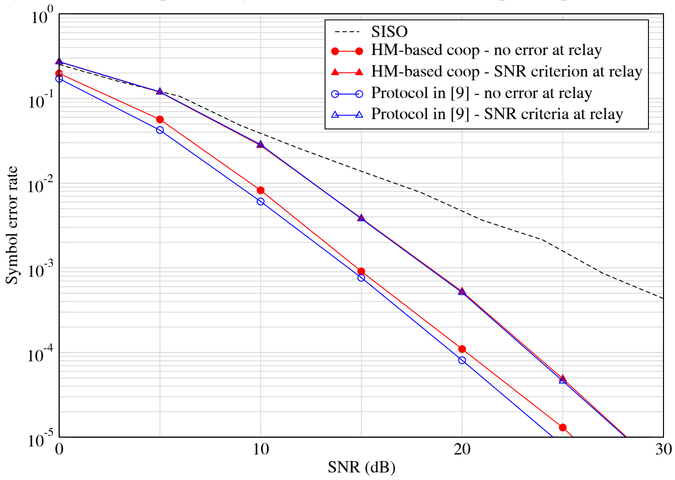

Fig. 7. Comparison with the symbol error rate of the HM-based cooperative scheme proposed in [9]. The modification of the protocol does not have a big impact on performance.

in the practical case of an SNR criterion at relay, SER curves are nearly superposed.

Finally, Fig. 8 shows the improvement in performance obtained by optimizing the parameter θ rather than using a 16-QAM, which corresponds to $\theta = 0.5$. One can observe a 1.5 dB improvement of the SER theoretical upper-bound, and a 0.7 dB improvement in the more realistic case of an SNR criterion.

VI. Conclusion

In this paper, we have studied the performance of a hierarchical modulation-based protocol which uses the distributed Alamouti code in order to provide diversity. We have introduced a modification of the protocol compared to the one exiting in literature, in order to facilitate the theoretical study of its performance. We have provided an upper-bound on its symbol error rate which is within 1 dB of the optimal performance of the protocol. Using this upper-bound, we have proposed a way to optimize the choice of the hierarchical

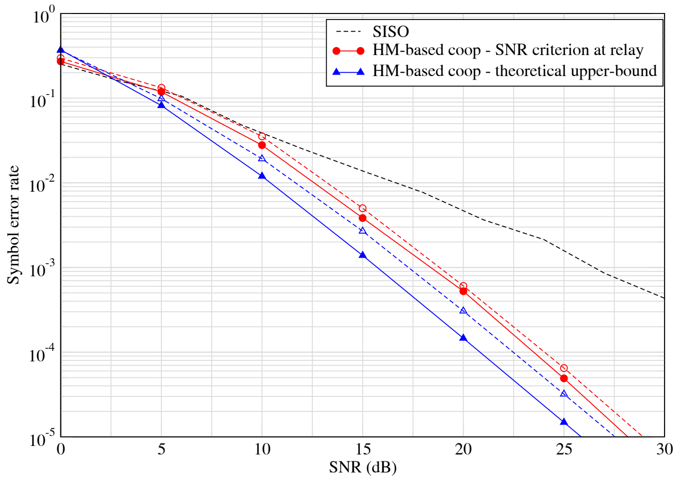

Fig. 8. Improvement in symbol error rate due to the optimization of θ: plain curves are obtained using the optimal θ^* and dashed ones using $\theta = 0.5$ (corresponding to a 16-QAM). A 1.5 dB improvement is obtained by optimizing θ.

modulation parameter, in order to improve the performance.

In a future work, this protocol could be generalized to larger networks with a higher number of relays, sources and destinations. The effect of a bad channel estimation could also be investigated and some solutions to attenuate this impact proposed.

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Appendix

A. Pairwise error probability over $\sqrt{|f|^2+|g|^2}$

1) If $\sigma_f^2 \neq \sigma_g^2$:

$$
\begin{aligned}
&E_h\left\{Q(\sqrt{|f|^2+|g|^2}A)\right\} \\
&= \int_0^\infty \int_0^\infty \left(\frac{1}{\sqrt{2\pi}}\int_{\sqrt{f^2+g^2}A}^\infty e^{-\frac{u^2}{2}}du\right)\frac{f}{\sigma_f^2}e^{-\frac{f^2}{2\sigma_f^2}}df\,\frac{g}{\sigma_g^2}e^{-\frac{g^2}{2\sigma_g^2}}dg \\
&= \frac{1}{\sqrt{2\pi}}\int_0^\infty\int_0^{\frac{u}{A}}\int_0^{\sqrt{\frac{u^2}{A^2}-g^2}}\frac{f}{\sigma_f^2}e^{-\frac{f^2}{2\sigma_f^2}}df\,\frac{g}{\sigma_g^2}e^{-\frac{g^2}{2\sigma_g^2}}dg e^{-\frac{u^2}{2}}du \\
&= \frac{1}{\sqrt{2\pi}}\int_0^\infty\int_0^{\frac{u}{A}}\left(1-e^{-\frac{u^2}{2A^2\sigma_f^2}}e^{-\frac{g^2}{2\sigma_f^2}}\right)\frac{g}{\sigma_g^2}e^{-\frac{g^2}{2\sigma_g^2}}dg e^{-\frac{u^2}{2}}du \\
&= \frac{1}{\sqrt{2\pi}}\int_0^\infty\left(1-e^{-\frac{u^2}{2A^2\sigma_g^2}}\right)e^{-\frac{u^2}{2}}du - \frac{1}{\sqrt{2\pi}}\int_0^\infty\left(\frac{\sigma_f^2}{\sigma_f^2-\sigma_g^2}\right. \\
&\qquad \left. -\frac{\sigma_f^2}{\sigma_f^2-\sigma_g^2}e^{-\frac{u^2}{2A^2}\left(\frac{1}{\sigma_g^2}-\frac{1}{\sigma_f^2}\right)}\right)e^{-\frac{u^2}{2A^2\sigma_f^2}}e^{-\frac{u^2}{2}}du \\
&= \frac{1}{2}-\frac{1}{2}\left(1-\frac{\sigma_f^2}{\sigma_f^2-\sigma_g^2}\right)\sqrt{\frac{A^2\sigma_g^2}{1+A^2\sigma_g^2}}-\frac{1}{2}\frac{\sigma_f^2}{\sigma_f^2-\sigma_g^2}\sqrt{\frac{A^2\sigma_f^2}{1+A^2\sigma_f^2}} \\
&= \frac{1}{2}\left(1-\frac{\sigma_f^2}{\sigma_f^2-\sigma_g^2}\sqrt{\frac{A^2\sigma_f^2}{1+A^2\sigma_f^2}}-\frac{\sigma_g^2}{\sigma_g^2-\sigma_f^2}\sqrt{\frac{A^2\sigma_g^2}{1+A^2\sigma_g^2}}\right)
\end{aligned}
$$

2) If $\sigma_f^2 = \sigma_g^2$:

$$
\begin{aligned}
&E_h\left\{Q(\sqrt{|f|^2+|g|^2}A)\right\}\\
&= \frac{1}{\sqrt{2\pi}}\int_0^\infty \left(1-e^{-\frac{u^2}{2A^2\sigma_f^2}}\right)e^{-\frac{u^2}{2}}du\\
&\qquad - \frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{u^2}{2A^2\sigma_f^2}e^{-\frac{u^2}{2A^2\sigma_f^2}}e^{-\frac{u^2}{2}}du\\
&= \frac{1}{2}-\frac{1}{2}\sqrt{\frac{A^2\sigma_f^2}{1+A^2\sigma_f^2}}-\frac{1}{\sqrt{2\pi}}\left(\left[-\frac{u^2}{2A^2\sigma_f^2}\frac{e^{-\frac{-u^2}{2}\left(1+\frac{1}{A^2\sigma_f^2}\right)}}{1+\frac{1}{A^2\sigma_f^2}}\right]_0^\infty\right.\\
&\qquad \left.+\int_0^\infty \frac{1}{2A^2\sigma_f^2}\frac{e^{-\frac{-u^2}{2}\left(1+\frac{1}{A^2\sigma_f^2}\right)}}{1+\frac{1}{A^2\sigma_f^2}}du\right)\\
&= \frac{1}{2}\left(1-\sqrt{\frac{A^2\sigma_f^2}{1+A^2\sigma_f^2}}-\frac{1}{2}\frac{1}{1+A^2\sigma_f^2}\sqrt{\frac{A^2\sigma_f^2}{1+A^2\sigma_f^2}}\right)
\end{aligned}
$$

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