Feasibility Conditions for the Existence of Walk-Regular Graphs

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ABSTRACT

A graph X is *walk-regular* if the vertex-deleted subgraphs of X all have the same characteristic polynomial. Examples of such graphs are vertex-transitive graphs and distance-regular graphs. We show that the usual feasibility conditions for the existence of a distance-regular graph with a given intersection array can be extended so that they apply to walk-regular graphs. Despite the greater generality, our proofs are more elementary than those usually given for distance-regular graphs. An application to the computation of vertex-transitive graphs is described.

1. INTRODUCTION

Let X be a finite undirected loop-free graph with vertex set $V=\{1,2,\dots,n\}$. The *adjacency matrix* of X is the $n\times n$ matrix $A=(a_{ij})$, where $a_{ij}=1$ if i and j are adjacent in X, and $a_{ij}=0$ otherwise.

For any matrix M, let σM denote the set of eigenvalues of M. If $\lambda\in\sigma M$, define $\mu_M(\lambda)$ to be the multiplicity of λ as an eigenvalue of M. If $\lambda\notin\sigma M$ it will be convenient to define $\mu_M(\lambda)=0$. The symbols M', $\operatorname{tr} M$, and M_{ij} denote the transpose, the trace, and the (i,j)th entry of M, respectively. The ith entry of a vector $\mathbf{x}_k$ will be written as $(\mathbf{x}_k)_i$.

A *partition* of V is a sequence $\pi=(V_1,V_2,\dots,V_m)$ of disjoint nonempty subsets of V whose union is V. The elements of π are known as its *cells*. Following Schwenk [8] we call π *equitable* if there are constants e_{ij} such that each vertex in cell V_i is adjacent to e_{ij} of the vertices in cell V_j $(1\leqslant i,j\leqslant m)$. The set of partitions of V which are equitable for X will be denoted by $\Pi(X)$, and the subset of those equitable partitions which have $\{v\}$ as their first cell will be denoted by $\Pi_v(X)$, for each $v\in V$.

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EXAMPLE 1.1. The discrete partition $(\{1\},\{2\},\dots,\{n\})$ is in $\Pi(X)$ for any X. The single-cell partition (V) is in $\Pi(X)$ if and only if X is regular.

EXAMPLE 1.2. Suppose X is distance-regular with diameter d (see Biggs [1]). Then the partition $(\{v\},D_1,D_2,\dots,D_d)$ is in $\Pi_v(X)$, where D_i contains those vertices at distance i from v $(1\leq i\leq d)$.

EXAMPLE 1.3. Let H be any subgroup of the automorphism group of X. Then a partition whose cells are the orbits of H is in $\Pi(X)$.

2. QUOTIENT MATRICES

If π is the equitable partition defined in Example 1.2, then the matrix (e_{ij}), or sometimes its transpose, is referred to as the *intersection array* corresponding to the distance-regular graph from which π is derived. Details of the theory of intersection arrays can be found in Biggs [1]. For our present purposes we will find it more convenient to use a related matrix suggested by Don Taylor, which we will call the *quotient matrix*. This has the considerable advantage of being symmetric.

Let $\pi=(V_1,V_2,\dots,V_m)$ be any partition of V. For $1\leq i\leq m$, define $k_i=|V_i|$. The $m\times n$ matrix $S=S(\pi)$ is defined by

$$\begin{aligned} S_{ij}&=k_i^{-1/2} && \text{if } j\in V_i,\\ &=0 && \text{otherwise.}\end{aligned}$$

Using S, we define the *quotient matrix* of X by π to be $Q=Q(X,\pi)=SAS'$, where A is the adjacency matrix of X.

The equitability of π can be defined algebraically via Q, as our first result shows.

THEOREM 2.1. *$\pi\in\Pi(X)$ if and only if $SA=QS$. Furthermore, if $\pi\in\Pi(X)$, then $Q_{ij}=(k_j/k_i)^{1/2}e_{ji}$ for $1\leq i,j\leq m$.*

Proof. For $v\in V$, $1\leq i\leq m$, define h_{vi} to be the number of vertices in V_i which are adjacent to v.

For $1\leq i\leq m$, $v\in V$, we have

$$(SA)_{iv}=k_i^{-1/2}\sum_{w\in V_i}A_{wv}=k_i^{-1/2}h_{vi} \tag{1}$$

and

$$(QS)_{iv} = \sum_{j=1}^{m} Q_{ij} S_{jv} = Q_{ij} k_j^{-1/2}, \tag{2}$$

where $v \in V_j$.

By comparing (1) and (2) we see that if $SA = QS$, then h_{vi} equals $(k_i/k_j)^{1/2} Q_{ij}$ for all $v \in V_j$, and so $\pi \in \Pi(X)$.

Conversely, if $\pi \in \Pi(X)$ and $1 \leqslant i, j \leqslant m$,

$$Q_{ij} = (SAS')_{ij} = \sum_{v=1}^{n} k_i^{-1/2} h_{vi} S_{jv}$$

$$= \sum_{v \in V_j} (k_i k_j)^{-1/2} e_{ji}$$

$$= (k_j / k_i)^{1/2} e_{ji},$$

which implies, as above, that $SA = QS$. ∎

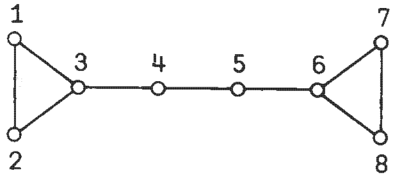
If π is equitable, there is a close relationship between the spectral properties of Q and A. Corollary 2.3 is very similar to a result of Haynsworth [4].

THEOREM 2.2. *Let $\pi \in \Pi(X)$. Then for any m-vector $\mathbf{x}$ and scalar λ, $Q\mathbf{x} = \lambda\mathbf{x}$ if and only if $A(S'\mathbf{x}) = \lambda(S'\mathbf{x})$.*

Proof. If $Q\mathbf{x} = \lambda\mathbf{x}$, then $S'Q\mathbf{x} = \lambda S'\mathbf{x}$ and so $AS'\mathbf{x} = \lambda S'\mathbf{x}$, by Theorem 2.1. If $AS'\mathbf{x} = \lambda S'\mathbf{x}$, then $SAS'\mathbf{x} = \lambda SS'\mathbf{x}$ and so $Q\mathbf{x} = \lambda\mathbf{x}$, since $SS' = I$. ∎

COROLLARY 2.3. *If $\pi \in \Pi(X)$, then $\mu_Q(\lambda) \leqslant \mu_A(\lambda)$ for all λ. Thus the characteristic polynomial of Q divides that of A.*

Proof. Suppose that $\lambda \in \sigma Q$. Then $\lambda \in \sigma A$ by the Theorem. Let $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_r\}$ be a full set of orthonormal eigenvectors of Q for λ. Then for $1 \leqslant i, j \leqslant r$ we find $(S'\mathbf{x}_i)'(S'\mathbf{x}_j) = \mathbf{x}_i' SS' \mathbf{x}_j = \mathbf{x}_i' \mathbf{x}_j$, since $SS' = I$. Hence, by the theorem, $\{S'\mathbf{x}_1, S'\mathbf{x}_2, \dots, S'\mathbf{x}_r\}$ is a set of orthonormal eigenvectors of A for λ. Therefore $\mu_Q(\lambda) \leqslant \mu_A(\lambda)$. ∎

X = 

$\pi=(\{3,6\},\{1,2,4,5,7,8\})$

$$S=\begin{bmatrix} 0 & 0 & 2^{-1/2} & 0 & 0 & 2^{-1/2} & 0 & 0 \\ 6^{-1/2} & 6^{-1/2} & 0 & 6^{-1/2} & 6^{-1/2} & 0 & 6^{-1/2} & 6^{-1/2} \end{bmatrix},$$

$$Q=\begin{bmatrix} 0 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}$$

FIG. 1.

EXAMPLE 2.4. In Fig. 1, we give an example of X, π, S and Q. The characteristic polynomial of Q is x^2-x-3, while that of A is

$$(x+1)^2(x-1)(x^2-x-3)(x^3-4x-1).$$

3. FEASIBILITY CONDITIONS

In this section we prove a strong condition on a sequence of matrices $Q_1, Q_2, \ldots, Q_n$ which is necessary for the existence of a graph X such that for each v, $Q_v=(X,\pi_v)$ for some $\pi_v \in \Pi_v(X)$. If the matrices Q_v are tridiagonal and all the same, our condition is equivalent to a well-known feasibility condition on the intersection array of a distance-regular graph (see Biggs [1]). However, despite the greater generality, we believe our proof to be more elementary.

LEMMA 3.1. *If* $\pi \in \Pi(X)$, *then* $Q^r = SA^rS'$ *for* $r=0,1,2,\ldots$.

Proof. By induction on r, using Theorem 2.1. ∎

COROLLARY 3.2. *If* $\pi \in \Pi_v(X)$, *then* $(A^r)_{vv} = (Q^r)_{11}$ *for* $v \in V$, $r=0,1,2,\ldots$

LEMMA 3.3. (a) $\operatorname{tr} A^r = \sum_{\lambda \in \sigma A} \mu_A(\lambda)\lambda^r$ *for* $r=0,1,2,\ldots$.

(b) *Let* M *be any real symmetric matrix. For each* $\lambda \in \sigma M$ *let* $\{\mathbf{x}_1(\lambda), \mathbf{x}_2(\lambda), \ldots, \mathbf{x}_{s_\lambda}(\lambda)\}$ *be a complete set of orthonormal eigenvectors of* M

for λ. *Then*

$$M^r = \sum_{\lambda \in \sigma M} \lambda^r \sum_{i=1}^{s_\lambda} \mathbf{x}_i(\lambda)\mathbf{x}_i(\lambda)' \qquad \textit{for} \quad r=0,1,2,\ldots.$$

Proof. Both claims are standard matrix-theory results. See Lancaster [5], for example. ∎

Let M be any real symmetric matrix. For each λ and i define the number $\theta(M,\lambda,i)$ as follows.

(a) If $\lambda \notin \sigma M$, define $\theta(M,\lambda,i)=0$ for all i.

(b) If $\lambda \in \sigma M$, let $\{x_1, x_2, \ldots, x_s\}$ be a full set of orthonormal eigenvectors of M for λ. Then define $\theta(M,\lambda,i)=\sum_{r=1}^{s}(x_r)_i^2$.

We are now in a position to prove our major theorem.

THEOREM 3.4. *Let X be any n-vertex graph with adjacency matrix A. For $v \in V$, let $\pi_v \in \Pi_v(X)$ and define $Q_v = Q(X,\pi_v)$.*
Then $\mu_A(\lambda)=\sum_{v=1}^{n}\theta(Q_v,\lambda,1)$, for any λ.

Proof. For $r=0,1,2,\ldots$

$$\begin{aligned}
\operatorname{tr} A^r &= \sum_{v=1}^{n} (Q_v^r)_{11}, && \text{by Corollary 3.2,}\\
&= \sum_{v=1}^{n} \sum_{\lambda \in \sigma Q_v} \lambda^r \theta(Q_v,\lambda,1), && \text{by Lemma 3.3,}\\
&= \sum_{v=1}^{n} \sum_{\lambda \in \sigma A} \lambda^r \theta(Q_v,\lambda,1), && \text{by Corollary 2.3.} \qquad (1)
\end{aligned}$$

Alternatively,

$$\operatorname{tr} A^r = \sum_{\lambda \in \sigma A} \mu_A(\lambda)\lambda^r, \qquad \text{by Lemma 3.3.} \qquad (2)$$

The Vandermonde matrix of order $|\sigma A|$ whose ith row contains the $(i-1)$th powers of the elements of σA is nonsingular. Hence the result follows on comparing (1) and (2). ∎

Since $\mu_A(\lambda)$ is a positive integer when $\lambda \in \sigma A$, the existence of X implies that the number $\sum_{v=1}^{n} \theta(Q_v, \lambda, 1)$ must be a positive integer whenever $\lambda \in \sigma Q_v$ for any v. This turns out to be a very strong condition on the sequence $Q_1, Q_2, \ldots, Q_n$, as we shall illustrate in Sec. 5. Meanwhile, we conclude this section with a few corollaries to Theorem 3.4.

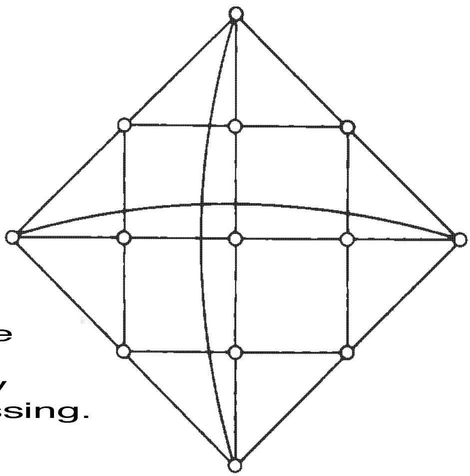
COROLLARY 3.5. *For each $\lambda \in \sigma A$ and $v \in V$, the number $\theta(Q_v, \lambda, 1)$ is independent of the choice of $\pi_v \in \Pi_v(X)$.*

COROLLARY 3.6. *For each $\lambda \in \sigma A$, there is at least one $v \in V$ such that for any $\pi_v \in \Pi_v(X)$ we have $\lambda \in \sigma Q(X, \pi_v)$.*

4. WALK-REGULAR GRAPHS

In this section we restrict our attention to a special class of graphs which allow us to apply Theorem 3.4 with only one quotient matrix instead of n.

A *closed walk* of length r in X is a sequence $v_0, v_1, \ldots, v_r$ of vertices of X such that $v_r = v_0$ and $\{v_{i-1}, v_i\}$ is an edge of X for $1 \leqslant i \leqslant r$. We say that X is *walk-regular* if, for each r, the number of closed walks of length r starting at v_0 is independent of the choice of v_0. One obvious class of walk-regular graphs is that of vertex-transitive graphs, those whose automorphism group is transitive on the vertices. Another family is that of distance-regular graphs [1], which includes that of strongly regular graphs [2]. An example which fits into neither of these classes is shown in Fig. 2.



Erratum: There is no vertex in the centre, only two edges crossing.

FIG. 2.

Since the number of closed walks of length 2 starting at v is just the degree of v, it is clear that a walk-regular graph is regular. Some of the other properties of walk-regular graphs can be found in the following theorems. The *characteristic polynomial* of a graph Y is the characteristic polynomial of its adjacency matrix, and is denoted by $\phi(Y)$.

THEOREM 4.1. *The following conditions are equivalent*:

(a) *X is walk-regular.*

(b) *For $r=1,2,\ldots$, the diagonal entries of A^r are all equal.*

(c) *For $v\in V$, let X_v be the subgraph of X formed by deleting vertex v. Then the graphs X_v have the same characteristic polynomial.*

(d) *For each $\lambda\in\sigma A$, $\theta(A,\lambda,i)$ is independent of i.*

Proof. The equivalence of (a) and (b) follows from the standard result that the ith diagonal entry of A^r is the number of closed walks of length r in X which start at i. The equivalence of (a) and (c) and of (a) and (d) follow from Lemmas 2.1 and 3.1 of [3]. ■

THEOREM 4.2. *X is walk-regular if and only if $\bar{X}$ is walk-regular.*

Proof. Let $\bar{A}$ be the adjacency matrix of $\bar{X}$, and let $\mathbf{c}$ be the n-vector with each entry equal to 1. Let k be the degree of X.

Then $\mathbf{c}$ is an eigenvector of both A and $\bar{A}$, corresponding to the eigenvalues k and $n-k-1$, respectively. If $\mathbf{y}$ is an eigenvector of A which is orthogonal to $\mathbf{c}$ and corresponds to an eigenvalue λ, then it is also an eigenvector of $\bar{A}$, corresponding to the eigenvalue $-1-\lambda$.

The result now follows by Theorem 4.1(d). ■

LEMMA 4.3. (a) *If Y is any graph with vertices $1,2,\ldots,r$, then $\phi'(Y)=\sum_{v=1}^{r}\phi(Y_v)$.*

(b) *If Y is a disconnected graph, with components $Y^1,Y^2,\ldots,Y^t$, then $\phi(Y)=\prod_{i=1}^{t}\phi(Y^i)$.*

Proof. The lemma follows readily from Exercises 4 and 6 on p. 50 of [5]. ■

THEOREM 4.4. *If X is disconnected, then it is walk-regular if and only if the components of X are walk-regular and have the same characteristic polynomial.*

Proof. Suppose that the components of X are walk-regular and have the same characteristic polynomial. Then the point-deleted subgraphs of each component have the same characteristic polynomial, by Lemma 4.3(a), and so X is walk-regular, by Lemma 4.3(b).

Suppose conversely that X is walk-regular, and has components $X^1, X^2, \ldots, X^t$. Then by Lemma 4.3, each X^i is walk-regular, and the products $\phi'(X^i)\prod_{j\neq i}\phi(X^j)$ are equal for $1 \leqslant i \leqslant t$. Therefore, the ratios $\phi'(X^i)/\phi(X^i)$ are equal, which implies that the polynomials $\phi(X^i)$ are equal, since they are monic. ■

THEOREM 4.5. *Let X and Y be walk-regular graphs. Then $X \times Y$ (cartesian product), $X \otimes Y$ (tensor product), $X * Y$ (strong product), and $X[Y]$ (lexicographic product) are walk-regular.*

Proof. Each case follows readily from Theorem 4.1(d), but we will omit the details. ■

From now on we will assume that X is walk-regular, with n vertices and degree k, and that A has s simple eigenvalues.

The next theorem generalizes one of Petersdorf and Sachs [7], who proved the same result for vertex-transitive graphs.

THEOREM 4.6. *Suppose that λ is a simple eigenvalue of A. Then λ is an integer of the form $k-2\alpha$ for $0 \leqslant \alpha \leqslant k$.*

Proof. Let $\mathbf{y}$ be the eigenvector of A which corresponds to λ. Then the entries of $\mathbf{y}$ have equal absolute value, by Theorem 4.1(d). The theorem now follows by considering the first row of the equation $A\mathbf{y} = \lambda\mathbf{y}$. ■

THEOREM 4.7. *n is even if $s \geqslant 2$, and is divisible by 4 if $s \geqslant 3$.*

Proof. Since X is regular, $\mathbf{c}$ is an eigenvector of A corresponding to the eigenvalue k. Suppose that λ is a simple eigenvalue other than k and that $\mathbf{y}$ is the corresponding eigenvector. Since the entries of $\mathbf{y}$ have equal absolute value by Theorem 4.1(d), and $\mathbf{y}$ is orthogonal to $\mathbf{c}$, n must be even.

Suppose that $\mathbf{z}$ is the eigenvector corresponding to a simple eigenvalue other than k or λ. Then, as before, the entries of $\mathbf{z}$ have equal absolute value. The mutual orthogonality of $\mathbf{c}$, $\mathbf{y}$, and $\mathbf{z}$ now implies that n is divisible by 4. ■

THEOREM 4.8. *If $n \geqslant 3$, then $s \leqslant n/2$.*

Proof. If n is odd, $s \leqslant 1$ by Theorem 4.7. Hence we assume n is even. If the eigenvalues of A are $\lambda_1 \leqslant \lambda_2 \leqslant \cdots \leqslant \lambda_{n-1} \leqslant k$, then those of $\bar{A}$ (the adjacency matrix of $\bar{X}$) are $-\lambda_{n-1}-1 \leqslant -\lambda_{n-2}-1 \leqslant \cdots \leqslant -\lambda_1 - 1 \leqslant n-k-1$. If either X or $\bar{X}$ is disconnected, then Theorem 4.4 implies that $s \leqslant 2$. If X and $\bar{X}$ are both connected, we see that $\bar{A}$ also has s simple eigenvalues. Applying Theorem 4.6 to X and to $\bar{X}$ gives us $s \leqslant k+1$ and $s \leqslant n-k$, which together imply $s \leqslant (n+1)/2$. Since we have assumed n to be even, $s \leqslant n/2$. ∎

When Theorem 3.4 is applied to a walk-regular graph, we find that only one quotient matrix is required.

THEOREM 4.9. *Let X be walk-regular. Then for $v \in V$ and any λ, $\mu_A(\lambda) = n\theta(Q_v, \lambda, 1)$.*

Proof. By Theorem 4.1(b),

$$\operatorname{tr} A^r = n(A^r)_{vv} = n(Q_v^r)_{11},$$

by Corollary 3.2, for $r = 0, 1, 2, \ldots$. The proof now follows that of Theorem 3.4. ∎

COROLLARY 4.10. *Let X be walk-regular, and suppose $\pi_v \in \Pi_v(X)$ for any v. Then $\sigma Q(X, \pi_v) = \sigma A$.* ∎

Our final result for this section generalizes a known result for symmetric graphs [1].

THEOREM 4.11. *Let X be a walk-regular graph with degree k, and suppose that for some v, the neighborhood of v in X is a single cell of π_v. Then the only possible simple eigenvalues of A are $-k$ and k.*

Proof. Let λ be a simple eigenvalue of A, and let $\mathbf{y}$ be the corresponding eigenvector. Then $\mathbf{y}$ is of the form $S(\pi_v)'\mathbf{x}$, by Theorem 2.2 and Corollary 4.10. This implies that the entries of $\mathbf{y}$ are constant on the neighborhood of v. The theorem now follows from the vth row of the equation $A\mathbf{y} = \lambda\mathbf{y}$. ∎

5. AN APPLICATION

The results of Sec. 4 have been used to advantage in the construction of every vertex-transitive graph with fewer than 20 vertices [6]. Suppose that X is a vertex-transitive graph with automorphism group G, and let H be any subgroup of the point-stabilizer G_1. As we claimed in Example 1.3, the partition π whose cells are the orbits of H is in $\Pi_1(X)$. The associated quotient matrix $Q = Q(X, \pi)$ must satisfy a number of conditions which we list in the following theorem.

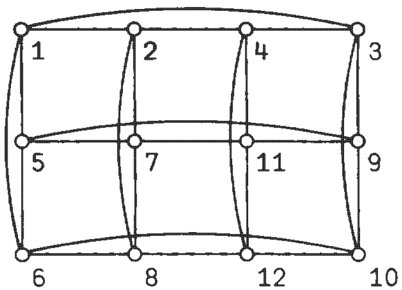
THEOREM 5.1. *For any λ, define $\mu(\lambda) = n\theta(Q, \lambda, 1)$.*

(a) *For each $\lambda \in \sigma Q$, $\mu(\lambda)$ is a positive integer.*
(b) *If $\mu(\lambda) = 1$, then λ is an integer.*
(c) *$\mu(\lambda) \geqslant \mu_Q(\lambda)$ for any λ.*
(d) *Suppose $\mu(\lambda) = 1$ for s values of $\lambda \in \sigma Q$. Then n is even if $s \geqslant 2$, and is divisible by 4 if $s \geqslant 3$.*

Proof. By Theorem 4.9, $\mu(\lambda) = \mu_A(\lambda)$ for any λ, which implies (a). Claims (b), (c), and (d) now follow immediately from Theorem 4.6, Corollary 2.3, and Theorem 4.7, respectively. ∎

The construction of vertex-transitive graphs involved many stages which it would not be appropriate to detail here. The only stage of interest to us at

$$\begin{bmatrix} 0 & \sqrt{2} & 0 & \sqrt{2} & 0 & 0 \\ \sqrt{2} & 0 & \sqrt{2} & 0 & \sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 & 0 & 0 & \sqrt{2} \\ \sqrt{2} & 0 & 0 & 1 & \sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 & \sqrt{2} & 1 & \sqrt{2} \\ 0 & 0 & \sqrt{2} & 0 & \sqrt{2} & 1 \end{bmatrix}$$



$$\pi = (\{1\}, \{2,3\}, \{4\}, \{5,6\}, \{7,8,9,10\}, \{11,12\})$$

FIG. 3.

the moment involved the application of Theorem 5.1 to 58,454 matrices, each of which was a candidate quotient matrix of some vertex-transitive graph. Since those graphs with regular automorphism groups ("GRRs") were constructed separately, it was found convenient to assume that H was a nontrivial p-group. Of the 58,454 matrices tested, only 709 satisfied all the requirements of Theorem 5.1. Conditions (a) and (b) eliminated many more cases than did (c) or (d). Out of the matrices which passed all four tests, 592 yielded one vertex-transitive graph each and 5 yielded two vertex-transitive graphs each. Many of the remaining 112 matrices probably correspond to nontransitive walk-regular graphs, but these were not detected by the program.

In Fig. 3 we give an example of Q and its unique vertex-transitive realisation.

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