A CORRECTION TO COLBOURN'S PAPER ON THE COMPLEXITY OF MATRIX SYMMETRIZABILITY

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Received November 1979; revised version received June 1980

Symmetrizable matrix, computational complexity, NP-complete, isomorphism complete

1. Introduction

A square matrix A is *transposable* if $A^T = PAQ$ for some permutation matrices P and Q, and *symmetrizable* if $RA = (RA)^T$ for some permutation matrix R. A recent paper [1] purports to show that the matrix symmetrizability problem is isomorphism complete, that is polynomial time equivalent to the graph isomorphism problem. Unfortunately, that paper is based on a misunderstanding about the contents of [4], as we shall indicate. In this note we show that the matrix transposability problem is isomorphism complete, whereas the matrix symmetrizability problem is NP-complete.

2. Symmetrizability versus transposability

Let $A = (a_{ij})$ be a square matrix of order n. From A we can construct an edge-labelled complete bipartite graph G(A). The vertices of G(A) are the rows and columns of A, and for $1 \leqslant i, j \leqslant n$ there is an edge from the i^{th} row to the j^{th} column, labelled with the entry a_{ij}.

Theorem 1 ([4]). A is transposable if and only if there is an automorphism of G(A) which interchanges the cells of the bipartition (i.e. the rows with the columns). A is symmetrizable if and only if there is such an automorphism of order two.

Examples are given in [2] and [4] of bipartite graphs having automorphisms which interchange the cells of the bipartition, but no such automorphisms of order two.

The condition for transposability given in Theorem 1 is stated in [1] as a condition for symmetrizability, and this incorrect result is then shown to imply that matrix symmetrizability is isomorphism complete. Fortunately, almost the same proof shows that matrix transposability is isomorphism complete.

Theorem 2. The matrix transposability problem is isomorphism complete.

The complexity of the matrix symmetrizability problem can be deduced from the following result, which was proved by Lalonde [2] as a consequence of a result of Lubiw [3].

Theorem 3 ([2]). Let G be a connected bipartite graph. Then the problem of determining whether G has an automorphism of order two interchanging the cells of the bipartition is NP-complete.

Corollary 1. The matrix symmetrizability problem is NP-complete.

Proof. The problem is obviously in NP. Furthermore, the problem of Theorem 3 can be reduced in polynomial time to a matrix symmetrizability problem by Theorem 1.

Finally, we note that Theorem 2 and Corollary 1 remain valid if restricted to matrices with 0–1 entries.

References

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